UNIT 1 – INTRODUCTION

2 MARK QUESTIONS AND ANSWERS

1. Define: Water Content ($w$)
   Water content is defined as the ratio of weight of water to the weight of solids in a given mass of soil.

2. Density of Soil: Define
   Density of soil is defined as the mass the soil per unit volume.

3. Bulk Density: Define ($\rho$)
   Bulk density is the total mass $M$ of the soil per unit of its total volume.

4. Dry Density: Define ($\rho_d$)
   The dry density is mass of soils per unit of total volume of the soil mass.

5. Define: Saturated Density ($\rho_{sat}$)
   When the soil mass is saturated, is bulk density is called saturated density

6. Define: Submerged Density ($\rho'$)
   The submerged density is the submerged mass of the soil solids per unit of total volume of the soil mass.

7. Define: Unit Weight of Soil Mass
   The unit mass weight of a soil mass is defined as it s weight per unit volume.

8. Bulk Unit Weight: Define ($\gamma$)
   The bulk weight is the total weight of a soil mass per unit of its total volume.

9. Dry Unit Weight: Define ($\gamma_d$)
   The dry unit weight isht weight of solids per unit of its total volume of the soil mass.

10. Unit Weight of Solids: Define ($\gamma_s$)
    The unit weight of soil solids is the weight of soil solids per unit volume of solids.
11. What Is Submerged Unit Weight ($\gamma'$)
   The submerged unit weight is the submerged weight of soil solids per unit of the total volume of soils.

12. What Is Saturated Unit Weight ($\gamma_{sat}$)
   Saturated unit weight is the ratio of the total weight of a saturated soil sample to its total sample.

13. What Is Void Ratio? (e)
   Void ratio of a given soil sample is the ratio of the volume of soil solids in the given soil mass.

14. What is Porosity? (n)
   The porosity of a given soil sample is the ratio of the volume of voids to the total volume of the given soil mass.

15. Degree of saturation: Define (Sr)
   The degree of saturation is defined as the ratio of the volume of water present in volume of water present in a given soil mass to the total volume of voids on it.

16. Define: percentage of air voids (n_a)
   Percentage of air voids is defined as the ratio of the volume of air voids to the total volume of soil mass.

17. Air content: Define (a_c)
   The air content is defined as the ratio of volume of air void to the volume of voids.

18. Define: Density Index ($I_D$) or Relative Compactive
   The density index is defined as the ratio of the differences between the voids ratio of the soil in the loosest state and its natural voids ratio ratio & to the differences between voids ratio in the loosest and densest states.

19. What is compaction?
   Compaction is a process by which the soil particles are artificially rearranged and packed together into a closer strata of contact by mechanical means in order to decrease the porosity (or voids ratio) of the soil and thus increase its dry density.

20. Aim of the compaction
   i) To increase the shear strength soil
   ii) To improve stability and bearing capacity
   iii) To reduce the compressibility
   iv) To reduce the permeability of the soil.
21. What are the methods available for sieve analysis?
   a) Dry sieve Analysis
   b) Wet sieve analysis

22. Atterberg limits: define

   The limit at which the soil, changes from one state to another state, is termed as atterberg limits.

23. Liquid limit: define

   Is the water content at which the soil, changes from liquid to plastic state liquid.

24. What is plastic limit?

   The maximum water content at which, soil changes from plastic to semi-solid state.

25. Distinguish between Residual and Transported soil. (AUC May/June 2012)

   Soils which are formed by weathering of rocks may remain in position at the place of region. In that case these are 'Residual Soils'. These may get transported from the place of origin by various agencies such as wind, water, ice, gravity, etc. In this case these are termed “Transported soil”. Residual soils differ very much from transported soils in their characteristics and engineering behaviour. The degree of disintegration may vary greatly throughout a residual soil mass and hence, only a gradual transition into rock is to be expected. An important characteristic of these soils is that the sizes of grains are not definite because of the partially disintegrated condition. The grains may break into smaller grains with the application of a little pressure.

26. Give the relation between $\gamma_{sat}$, G, $\gamma_w$ and e. (AUC May/June 2012)

   $\gamma_{sat} = G \gamma_o (1-n) + \gamma'_o . n$  

   $\gamma_{sat} = \frac{w_{sat}}{V}$  

   $\gamma_{sat} = \frac{(W_d + W_w)}{V}$  

   $\gamma_{sat} = \frac{\gamma_s V_s + \gamma_w V_w}{V}$

   From, fig (ii)

   $V_s = 1$, $V_w = e$, and $V = 1+e$

   $\gamma_{sat} = \frac{\gamma_s.1 + \gamma_w e}{1+e}$
\[
\gamma_{w} = \frac{G\gamma_{w} + \gamma_{w}e}{1 + e}
\]

\[
\gamma_{sat} = \frac{(G + e)\gamma_{w}}{1 + e}
\]

From fig (ii)

\[V_{s} = 1 - n, \ V_{w} = n, \ V = 1\]

\[
\gamma_{sat} = \frac{\gamma_{s}(1 - n) + \gamma_{w}n}{1}
\]

27. A compacted sample of soil with a bulk unit weight of 19.62 kN/m\(^3\) has a water content of 15 per cent. What are its dry unit weight, degree of saturation, ?
Assume \(G = 2.65\).

\[(AUC \ Apr/May \ 2010)\]

Step 1: To find the dry density

\[
\gamma_{d} = \frac{\gamma_{s}}{1 + w} = \frac{19.62}{1 + 0.15} = 17.06 \ \text{KN/m}^2
\]

Step : To find the Void ratio

\[
e = \frac{G \times \gamma_{w}}{\gamma_{d}} - 1 = \frac{2.65 \times 9.81}{17.06} - 1
\]

\[e = 2.68\]

Step 3: To find the Degree of saturation

\[
S = \frac{wG}{e} = \frac{0.15 \times 2.65}{2.68} = 0.14\%
\]
Step 2: To find the porosity

\[
n = \frac{e}{1 + e} = \frac{0.68}{1 + 2.68} = 0.72\%
\]

28. What are all the Atterberg limits for soil and why it is necessary? (AUC Nov/Dec 2012)

The consistency limits or Atterberg limits and certain indices related to these may be defined as follows:

**Liquid Limit**

‘Liquid limit’ (LL or w_L) is defined as the arbitrary limit of water content at which the soil is just about to pass from the plastic state into the liquid state. At this limit, the soil possesses a small value of shear strength, losing its ability to flow as a liquid. In other words, the liquid limit is the minimum moisture content at which the soil tends to flow as a liquid.

**Plastic Limit**

‘Plastic limit’ (PL or w_p) is the arbitrary limit of water content at which the soil tends to pass from the plastic state to the semi-solid state of consistency. Thus, this is the minimum water content at which the change in shape of the soil is accompanied by visible cracks, i.e., when worked upon, the soil crumbles.

**Shrinkage Limit**

‘Shrinkage limit’ (SL or w_s) is the arbitrary limit of water content at which the soil tends to pass from the semi-solid to the solid state. It is that water content at which a soil, regardless, of further drying, remains constant in volume. In other words, it is the maximum water content at which further reduction in water content will not cause a decrease in volume of the soil mass, the loss in moisture being mostly compensated by entry of air into the void space. In fact, it is the lowest water content at which the soil can still be completely saturated. The change in colour upon drying of the soil, from dark to light also indicates the reaching of shrinkage limit.

Upon further drying, the soil will be in a partially saturated solid state; and ultimately, the soil will reach a perfectly dry state.

**Plasticity Index**

‘Plasticity index’ (PI or I_p) is the range of water content within which the soil exhibits plastic properties; that is, it is the difference between liquid and plastic limits.
\[ PI(\text{or } I_p) = (LL - PL) = (w_L - w_L) \]

When the plastic limit cannot be determined, the material is said to be non-plastic (NP). Plasticity index for sands is zero.

**Shrinkage Index**

'Shrinkage index' \((SI \text{ or } I_s)\) is defined as the difference between the plastic and shrinkage limits of a soil; in other words, it is the range of water content within which a soil is in a semi-solid state of consistency.

\[ SI(\text{or } I_s) = (PL - SL) = (w_p - w_s) \]

**Consistency Index**

'Consistency index' or 'Relative consistency' \((CI \text{ or } I_c)\) is defined as the ratio of the difference between liquid limit and the natural water content to the plasticity index of a soil:

\[ CI(\text{or } I_c) = \frac{(LL - w)}{PI} = \frac{(w_L - w)}{I_p} \]

where \(w\) = natural water content of the soil (water content of a soil in the undisturbed condition in the ground).

- If \(I_c = 0\), \(w = LL\)
- \(I_c = 1\), \(w = PL\)
- \(I_c > 1\), the soil is in semi-solid state and is stiff.
- \(I_c < 0\), the natural water content is greater than \(LL\), and the soil behaves like a liquid.

**Liquidity Index**

'Liquidity index \((LI \text{ or } I_L)\)' or 'Water-plasticity ratio' is the ratio of the difference between the natural water content and the plastic limit to the plasticity index:

\[ LI(\text{or } I_L) = \frac{(w - PL)}{PI(\text{or } I_p)} = \frac{w - w_p}{I_p} \]

- If \(I_L = 0\), \(w = PL\)
- \(I_L = 1\), \(w = LL\)
- \(I_L > 1\), the soil is in liquid state.
29. Define sieve analysis and sedimentation analysis and what is the necessity of these two analysis?  

This classification test determines the range of sizes of particles in the soil and the percentage of particles in each of these size ranges. This is also called ‘grain-size distribution’; ‘mechanical analysis’ means the separation of a soil into its different size fractions.*

The particle-size distribution is found in two stages:

(i) Sieve analysis, for the coarse fraction.

(ii) Sedimentation analysis or wet analysis, for the fine fraction.

‘Sieving’ is the most direct method for determining particle sizes, but there are practical lower limits to sieve openings that can be used for soils. This lower limit is approximately at the smallest size attributed to sand particles (75μ or 0.075 mm).

Sieving is a screening process in which coarser fractions of soil are separated by means of a series of graded mesh. Mechanical analysis is one of the oldest test methods for soils.

30. What is a zero air voids line or saturated line? Draw a compaction curve and show the zero air voids line.

A line showing the relation between water content and dry density at a constant degree of saturation S may be established from the equation:

\[
\gamma_d = \frac{G_{lw}}{1 + \frac{wG}{S}}
\]

Figure of compaction curve

\[
\gamma_d = \frac{G_{lw}}{1 + \frac{wG}{100}}
\] for this situation.
31. What is porosity of a given soil sample? (AUC Apr / May 2011)

‘Porosity’ of a soil mass is the ratio of the volume of voids to the total volume of the soil mass. It is denoted by the letter symbol \( n \) and is commonly expressed as a percentage:

\[
n = \frac{V_v}{V} \times 100
\]

Here

\[V_v = V_a + V_w ; V = V_a + V_w + V_s\]

32. What is water content in given mass of soil? (AUC Apr / May 2011)

‘Water content’ or ‘Moisture content’ of a soil mass is defined as the ratio of the weight of water to the weight of solids (dry weight) of the soil mass. It is denoted by the letter symbol \( w \) and is commonly expressed as a percentage:

\[
w = \frac{W_w}{W_s \text{ or } W_d} \times 100
\]

\[= \frac{(W - W_d)}{W_d} \times 100\]

33. Define :

(a) Porosity

(b) Void ratio. (AUC Nov/Dec 2010)

(c)

Porosity

‘Porosity’ of a soil mass is the ratio of the volume of voids to the total volume of the soil mass. It is denoted by the letter symbol \( n \) and is commonly expressed as a percentage:

\[
n = \frac{V_v}{V} \times 100
\]

Here

\[V_v = V_a + V_w ; V = V_a + V_w + V_s\]

Void ratio.

‘Void ratio’ of a soil mass is defined as the ratio of the volume of voids to the volume of solids in the soil mass. It is denoted by the letter symbol \( e \) and is generally expressed as a decimal fraction:

\[
e = \frac{V_v}{V_s}
\]

Here

\[V_v = V_a + V_w\]
16 MARK QUESTIONS AND ANSWERS

1. Writes notes on nature of soil?

a) The stress strain relation ship for a soil deposit is nonlinear . hence the difficulty in using easily determinable parameter to describe its behavior.

b) Soil deposits have a memory for stress they have undergone in their geological history. Their behavior is vastly influenced by their stress history; time and environment are other factors which may alter their behavior.

c) Soil deposit being for from homogenous , exhibit properties which vary from location

d) As soil layers are buried and hidden from view. One has to rely on test carried out on small samples that can be taken; there is no grante that the soil parameters are truly representative of the field strata

e) No sample is truly undisturbed. in a soil which is sensitive to disturbance; the behavior submersed from the laboratory tests may not reflect the likely behavior of the field stratum.

2. Explain the problems related to soils.

Soils is, the ultimate foundation material which supports the structure the proper functioning of the structure will, therefore, depend critically element resting on the subsoil. Here the term foundation is used in the conventional sense. A substructure that distributes the load to the ultimate foundation, namely, the soil.

From ancient times, man has used soil for the construction of tombs, monuments, dwellings and barrages for storing water. In the design and construction of underground structures such as tunnels, conduits, power houses, bracings for excavations and earth retaining structure, the role of soil is again very crucial, since the soil is in direct contact with the structure, it acts as a medium of load transfer and hence for any analysis of forces acting on such structure, one has to consider the aspects of stress distribution through the soil.

The structure, two causes stresses and strain in the soils, while the stability of the structure itself is affected by soil behavior. The class problems where the structure and soil mutually interact are known as soil-structure interaction problems. There are a host of other civil engineering problems related to soils. For designing foundations for machines such as turbine, compressors, forges etc…. which transmit vibrations to the foundation soil, one has to understand the behavior of soil under vibratory loads.

The effect of quarry blasts, earthquakes and nuclear explosions on structures is greatly influenced by the soil medium through which the shock waves traverse. In these parts of the world which experience freezing temperatures, problems arise because the soil expand upon freezing and exert a force on the structure in the contact with them.

Thawing (due to melting ice) of the soil results in a soil results in a loss of strength in the soil. Structure resting on these soils will perform satisfactorily only if
measures are taken to prevent frost heave or designed to withstand the effects of the freezing and thawing

3. Prove that: \( e = \frac{wG}{S_r} \)

Soil element in terms of \( e_w \) and \( e \)

From figure: \( e_w \) -- volume of water
\( e \) -- Volume of voids
\( G \) - specific gravity

The volume of solids is equal to unity

\[
S_r = \frac{V_w}{V_v} = \frac{e_w}{e}
\]

\( e_w = e S_r \rightarrow (1) \)

\( e_w = e, \) when fully saturated sample.

\[
w = \frac{W \gamma_s}{W_d} \frac{e_w \gamma_w}{\gamma_s} = \frac{e_w}{1}
\]

\[
G = \frac{\gamma_s}{\gamma_w} \rightarrow \gamma = G \gamma_w
\]

\[
w = \frac{e_w \gamma_w}{G \gamma_w} = \frac{e_w}{G}
\]

\( e_w = w.G \rightarrow (2) \)

From equation (1) & (2)

\[
e = \frac{wG}{S_r} \rightarrow (3)
\]

When fully saturated sample, \( S_r = 1 \) and \( w = w_{sat} \)

\( e = w_{sat} G \)
4. Prove that:

\[ n_a = \frac{e(1 - S_r)}{1 + e} \]

\[ n_a = \frac{V_a}{V} \]

\[ V_a = Vv - V_w - e \cdot e_w \]

\[ V = Vs + Vv = 1 + e \]

\[ n_a = \frac{e - e_w}{1 + e} \]

\[ e_w = e \cdot Sr \quad \text{from equ} \quad \rightarrow (1) \]

\[ n_a = e \cdot e \cdot Sr / (1 + e) = e (1 - Sr) / (1 + e) \quad \rightarrow (4) \]

5. Prove that:

\[ n_a = n \ a_c \]

\[ a_c = \frac{v_a}{v_v} ; ; ; \quad n = \frac{v_v}{v} \]

\[ n_a = \frac{v_a}{v_v} = n \ a_c \quad \rightarrow (5) \]

6. Prove that:

\[ \gamma_d = \frac{G \gamma_w}{1 + e} \]

\[ \gamma_d = \frac{wd_v}{V} \]

\[ \gamma_d = \frac{\gamma \cdot v_i}{V} \]
Vs = 1 (refer soil element in terms of ‘e’ figure)

V = (1 + e)

\( \gamma_d = \frac{\gamma_s \cdot 1}{1 + e} \)

\( \gamma_s = G \cdot \gamma_w \)

\( \gamma_d = \frac{G \gamma_w}{1 + e} \rightarrow (6) \)

Note:

\( 1 + e = \frac{G \gamma_w}{1 + e} \)

\( e = \frac{G \gamma_w}{1 + e} - 1 \)

From figure (ii) soil element in terms of ‘n’

\( Vs = \frac{G \gamma_w (1 - n)}{1} = (1 - n) G \gamma_w \rightarrow (7) \)

7. a. Prove that:

\( \gamma_{sat} = G \gamma_w (1 - n) + \gamma_w \cdot n \)

\( \gamma_{sat} = \frac{w_{sat}}{V} \)

\[ = \left( \frac{W_d + W_w}{V} \right) \]

\[ = \frac{\gamma_s V_s + \gamma_w V_w}{V} \]

From, fig (ii)

Vs = 1, Vw = e, and V = 1 + e

\( \gamma_{sat} = \frac{\gamma_s \cdot 1 + \gamma_w e}{1 + e} \)

\[ = \frac{G \gamma_w + \gamma_w \cdot e}{1 + e} \]
\[ \gamma_{sat} = \frac{(G + e) \gamma_w}{1 + e} \rightarrow (8) \]

From fig (ii)

\[ V_s = 1-n, \ V_w = n, \ V = 1 \]

\[ \gamma_{sat} = \frac{\gamma_s(1-n) + \gamma_w n}{1} \rightarrow (9) \]

7. b. Prove that:

\[ \gamma = \frac{(G + e S_r) \gamma_w}{1 + e} \]

\[ \gamma = \frac{V_s V + \gamma_w V}{V} \]

Refer: figure (iii)

\[ V_s = 1, \ V_w = e, \text{ and } V = 1 + e \]

\[ \gamma = \frac{\gamma_s 1 + \gamma_w e}{1 + e} \]

\[ \gamma_s = G \ \gamma_w \quad \& \quad \gamma_w = e S_r \]

\[ \gamma = \frac{G \gamma_w + \gamma_w e S_r}{1 + e} \rightarrow (10) \]

If the soil is perfectly dry, \( S_r = 0 \)

When \( S_r = 1 \)

\[ \gamma \text{ Become } \gamma_{sat} = \frac{(G + e) \gamma_w}{1 + e} \rightarrow (11) \]
8. a. Prove that: 
\[ \gamma' = \frac{(G-1)\gamma_w}{1+e} \]
\[ \gamma' = \gamma_{sat} - \gamma_w \]
\[ = \frac{(G+e)\gamma_w - \gamma_w}{1+e} \]
\[ = \frac{(G-1)\gamma_w}{1+e} \rightarrow (12) \]

8. b. Prove that: 
\[ \gamma_d = \frac{\gamma}{1+w} \]

Water content: \( w = \frac{W_w}{W_d} \)

\[ 1 + w = \frac{W_w + W_d}{W_d} = \frac{W}{W_d} \]

\( W_d = \frac{W}{1+w} \rightarrow (13) \)

\[ \gamma_d = \frac{W_d}{V} = \frac{W}{(1+w)V} \]

\[ \gamma_d = \frac{\gamma}{1+w} \rightarrow (!4) \]

9. Prove that:
\[ \gamma' = \gamma_d - (1-n) \gamma_w \]

From fig (iii)
\[ (W_d)_{sub} = 1 \cdot \gamma_s - 1 \cdot \gamma_w \]
\[ (W_d)_{sub} = (G-1) \gamma_w \]

\[ V = 1+e \]

From equation (12)
\[ \gamma' = (W_d)_{sub} \cdot V \]
\[ \frac{(G-1)\gamma_w}{1+e} = \frac{G\gamma_w}{1+e} - \frac{\gamma_w}{1+e} \]

From equation (6)

\[ \gamma_d = \frac{G\gamma_w}{1+e} \]

\[ 1 / (1+e) = 1-n \]

\[ \gamma' = \gamma_d - (1-n) \gamma_w \rightarrow (15) \]

10.a. Prove that

\[ \gamma = \gamma_d + Sr(\gamma_{sat} - \gamma_d) \]

From equation (10)

\[ \gamma = \frac{G\gamma_w + \gamma_w eSr}{1+e} \]

\[ = \frac{G\gamma_w + \gamma_w eSr}{1+e} \]

\[ = \gamma_d + Sr \left[ \frac{(G+e)\gamma_w}{1+e}, \frac{G\gamma_w}{1+e} \right] \]

\[ = \gamma_d + Sr ((\gamma_{sat} - \gamma_d) \rightarrow (16) \]

10.b. Prove that:

\[ \gamma_d = \frac{G\gamma_w}{1+w_{sat}G} \]

From equation (6)

\[ \gamma_d = \frac{G\gamma_w}{1+e} \]

From equation (3)
\[ \gamma_d = \frac{G \gamma_w}{1 + \frac{w}{Sr}} \rightarrow (17) \]

When \( Sr = 1 \), \( \gamma_d = \frac{G \gamma_w}{1 + W_{sat} G} \rightarrow (17.a) \)

11. Prove that

\[ \gamma_d = \frac{(1-n)G \gamma_w}{1 + wG} \]

From fig (i)

\[ V = V_a + V_w + V_s \]

\[ = V_a + W_w / \gamma_w + W_d / \gamma_s \]

\[ 1 = \frac{V_a}{V} + \frac{w.W_d}{\gamma_w.V} + \frac{W_d}{\gamma_s.V} \]

\[ 1 = \frac{V_a}{V} + \frac{w.\gamma_d}{\gamma_w} + \frac{\gamma_d}{\gamma_s} \]

\[ 1 - \frac{V_a}{V} = \frac{w.\gamma_d}{\gamma_w} + \frac{\gamma_d}{G \gamma_w} \]

\[ 1 - n_a = \frac{\gamma_d}{\gamma_w} \left( w + \frac{1}{G} \right) \]

\[ \gamma_d = \frac{(1-n_a)G \gamma_w}{(1 + wG)} \]

12. A soil sample has a porosity of 40\% . The specific gravity of solids 2.70. Calculate

(a) void ratio
(b) Dry density
(c) Unit weight if the soil is 50\% saturated
(d) Unit weight if the soil is completely saturated

Solution:

(a) \( e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667 \)
(b) \( \gamma_d = \frac{G \gamma_w}{1 + e} = \frac{2.7 \times 9.81}{1 + 0.667} = 15.89 \text{ KN/m}^3 \)

(c) \( e = \frac{wG}{Sr} \)

\[
w = \frac{eSr}{G} = \frac{0.667 \times 0.5}{2.7} = 0.124
\]

\( \gamma = \gamma_d (1 + w) = 15.89 \times 1.124 = 17.85 \text{ KN/m}^3 \)

(d) When the soil is fully saturated

\( e = w_{sat} G \)

\( w_{sat} = \frac{e}{G} = \frac{0.667}{2.7} = 0.247 \)

\( \gamma_{sat} = G \gamma_w (1 - n) + \gamma_w .n \)

\( = 2.7 \times 9.81(1 - 0.4) + 9.81 \times 0.4 = 19.81 \text{ KN/m}^3 \)

13. An undisturbed sample of soil has a volume of 100 cm\(^3\) and mass of 190 g. On oven drying for 24 hrs, the mass is reduced to 160 g. If the specific gravity grain is 2.68, determine the water content, voids ratio and degree of saturation of the soil.

**Solution:**

\( M_w = 190 - 160 = 30 \text{ g} \)

\( M_d = 160 \text{ g} \)

\( W = \frac{M_w}{M_d} = \frac{30}{160} = 0.188 = 18.8\% \)

Mass of moist soil = \( M = 190 \text{ g} \)

Bulk density = \( \frac{M}{V} = 190 / 100 = 1.9 \text{ g/cm}^3 \)

\( \gamma = 9.81 \times \rho = 9.81 \times 1.9 = 18.64 \text{ KN/m}^3 \)

\( \gamma_d = \frac{\gamma}{1 + w} \)

\( = \frac{18.64}{1 + 0.188} = 15.69 \text{ KN/m}^3 \)
\[ e = \frac{G \gamma_w}{\gamma_d} - 1 \]

\[ = \frac{2.68 \times 9.81}{15.69} - 1 = 0.67 \]

\[ Sr = \frac{wG}{e} = \frac{0.188 \times 2.68}{0.67} = 0.744 = 74.45\% \]

14. The in-situ density of an embankment, compacted at a water content of 12% was determined with the help of core cutter. The empty mass of the cutter was 1286 g and the cutter full of soil had a mass of 3195 g, the volume of the cutter being 1000 cm\(^3\). Determine the bulk density, dry density and the degree of saturation of the embankment.

If the embankment becomes fully saturated during rains, what would be its water content and saturated unit weight? Assume no volume change in soil on saturation. Take the specific gravity of the soil as 2.70.

Solution:

Mass of soil in cutter

\[ M = 3195 - 1286 = 1909 \text{ g} \]

Bulk density \( \rho = M / V = 1909 / 1000 = 1.909 \text{ g/cm}^3 \)

Bulk unit weight \( \gamma = 9.81 \times \rho \)

\[ = 9.81 \times 1.909 = 18.73 \text{ KN/m}^3 \]

\[ \gamma_d = \frac{\gamma}{1 + w} = \frac{18.73}{1 + 0.12} = 16.72 \text{ KN/m}^3 \]

\[ e = \frac{G \gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.81}{16.72} - 1 = 0.584 \]

\[ Sr = \frac{wG}{e} = \frac{0.12 \times 2.7}{0.584} = 0.555 = 55.5\% \]

At saturation:

Since the volume remains the same, the voids ratio also remains unchanged.

\[ e = \frac{w_{sat}G}{w_{sat}G} = \frac{0.584}{2.7} = 0.216 = 21.6\% \]
\[ \gamma_{sat} = \frac{(G + e)\gamma_w}{1 + e} = \frac{(2.7 + .584)9.81}{1 + .584} = 20.34 \text{ KN/ m}^3 \]

15. The in-situ percentage voids a sand deposit is 34 percent. For determining the density index, dried sand from the stratum was first filled loosely in a 1000 cm\(^3\) mould and was then vibrated to give a maximum density. The loose dry mass in the mould was m1610 g and dense dry mass at maximum compaction was found to be 1980 g. Determine the density index if the specific gravity of the sand particles 2.67

Solution

\[ n = 34\% \]
\[ e = n \div (1 - n) = 0.34 \div (1 - 0.34) = 0.515 \]

\[ (\gamma_d)_{\text{max}} = \frac{1980}{1000} \times 9.81 = 19.42 \text{ KN/ m}^3 \]
\[ (\gamma_d)_{\text{min}} = \frac{1610}{1000} \times 9.81 = 15.79 \text{ KN/ m}^3 \]

\[ e_{\text{min}} = \frac{G\gamma_w}{(\gamma_d)_{\text{min}}} - 1 = \frac{2.67 \times 9.81}{19.42} - 1 = 0.349 \]
\[ e_{\text{max}} = \frac{G\gamma_w}{(\gamma_d)_{\text{max}}} - 1 = \frac{2.67 \times 9.81}{15.79} - 1 = 0.659 \]

\[ I_D = (e_{\text{max}} - e) / (e_{\text{max}} - e_{\text{min}}) = (0.659 - 0.515) / (0.659 - 0.349) \]
\[ = 0.465 = 46.5\% \]

16. The mass specific gravity (apparent gravity) of a soil equals 1.64. The specific gravity of solids is 2.70. Determine the voids ratio under assumption that the soil is perfectly dry. What would be the voids ratio, if the sample is assumed to have a water content of 8 percent?

Solution:

When the sample is dry
\[ G_m = \frac{\gamma_d}{\gamma_w} = 1.64 \]
\[ \gamma_d = 1.64 \times \gamma_w = 1.64 \times 9.81 = 16.09 \text{ KN/ m}^3 \]
\[ e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.81}{16.09} - 1 = 0.646 \]
When the sample has water content

\[ w = 8\% \]

\[ \gamma = 1.64 \times \gamma_w = 1.64 \times 9.81 = 16.09 \text{ KN/m}^3 \]

\[ \frac{\gamma - \gamma}{1 + w} = \frac{16.09}{1 + 0.08} = 14.9 \text{ KN/m}^3 \]

\[ e = \frac{G \gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.81}{14.9} - 1 = 0.78 \]

17. A natural soil deposit has a bulk unit weight of 18.44 KN/m³, water content of 5 %. Calculate the amount of water required to the added to 1 m³ of soil to raise the water content to 15 %. Assume the void ratio to remain constant. What will then be the degree of saturation? Assume G= 2.67

Solution:

\[ \gamma = 18.44 \text{ KN/m}^3, w = 5\% \]

\[ \gamma_d = \frac{\gamma}{1 + w} = \frac{18.44}{1 + 0.05} = 17.56 \text{ KN/m}^3 \]

\[ w = \frac{W_w}{W_d} = 0.05 \]

For one cubic meter of soil, \( v = 1 \text{ m}^3 \)

\[ W_d = \gamma_d \times V = 17.56 \times 1 = 17.56 \text{ KN.} \]

\[ W_w = 0.05 \times W_d = 0.05 \times 17.56 = 0.878 \text{ KN} \]

\[ V_w = \frac{W_w}{\gamma_d} = \frac{0.878}{9.81} = 0.0895 \text{ m}^3 \]

Later, when \( w = 15\% \)

\[ W_w = w \times W_d = 0.15 \times 17.56 = 2.634 \text{ KN} \]

\[ V_w = \frac{W_w}{\gamma_w} = \frac{2.534}{9.81} = 0.2685 \text{ m}^3 \]

Hence additional water required to raise the water content from 5 % to 15 % \( = 0.2685 - 0.0895 = 0.179 \text{ m} = 179 \text{ liters.} \)

\[ \text{Voids ratio, } e = \frac{G \gamma_w}{\gamma_d} - 1 = \frac{2.67 \times 9.81}{17.56} - 1 = 0.49 \]
After the water has been added, ‘e’ remains the same

\[ Sr = \frac{w \cdot G}{e} = 0.15 \times 2.67 / 0.49 = 0.817 = 81.7\% \]

18. Calculate the unit weighs and specific gravities of solids of (a) soil composed of pure quartz and (b) a soil composed of 60% quartz, 25% mica, and 15% iron oxide. Assume that both soils are saturated and have voids of 0.63. Take average and for iron oxide = 3.8

Solution:

a) For the soil composed of pure Quartz,

\[ G \text{ for quartz} = 2.66 \]

\[ \gamma_{sat} = \frac{(G + e) \gamma_w}{1 + e} = \frac{(2.66 + 0.63) \times 9.81}{1 + 0.63} = 19.8 \text{ KN/m}^3 \]

b) For the composite soil,

\[ G \text{ average} = (2.66 \times 0.6) + (3.0 \times 0.25) + (3.8 \times 0.15) \]

\[ = 1.6 + 0.75 + 0.57 = 2.92 \]

\[ \gamma_{sat} = \frac{(G + e) \gamma_w}{1 + e} = \frac{(2.92 + 0.63) \times 9.81}{1 + 0.63} = 21.36 \text{ KN/m}^3 \]

19. A soil has a bulk unit weight of 20.22 KN/m\(^3\) and water content of 15%. Calculate the water content if the soil partially dries to a unit weight of 19.42 KN/m\(^3\) and voids ratio remains unchanged.

Solution:

Before drying,

\[ \gamma = 20.11 \text{ KN/m}^3 \]

\[ \gamma_d = \frac{20.11}{1 + 0.15} = 17.49 \text{ KN/m}^3 \]

Since after drying, 'e' does not change, 'V' and \(\gamma_d\) are the same,

\[ \gamma - \gamma_d (1 + w) \]

\[ 1 + w = \frac{\gamma}{\gamma_d} = 19.42 / 17.49 = 1.11 \]

\[ w = 1.11 - 1 = 11\% \]
20. A cube of dried clay having sides 4 cm long has a mass of 110 g. The same cubes of soil, when saturated at unchanged volume, has mass of 135 g. Draw the soil element showing the volumes and weights of the constituents, and then determine the specific gravity of soil solids and voids ratio.

Solution:

Volume of soil = (4)³ = 64 m³
Mass water after saturation
= 135 - 110 = 25 g

Volume of solids = 25 cm³
Volume of solids = 64 - 25 = 39 m³
Ms = 110 g

\[
G = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s}{\rho_w} = \frac{Ms}{Vs} = \frac{110}{39} = 2.82
\]

\[
e = \frac{Vs}{Vd} = \frac{25}{39} = 0.64
\]

21. a. Explain Dry sieve analysis

The soil should be oven-dry, it shouldn’t contain any lump, if necessary, it should be pulverized. If organic matters in the soil;, it taken air – dry instead of oven dry. The sample is sieved through a 4.75 mm IS sieve .the portion retained on the sieve is gravel fraction or plus 4.75 mm material .then gravel fraction is sieved through the set of coarse sieves manually or mechanical shaker.

The minus 4.75mm fraction is sieves through the set if fine sieves .the sample is placed in the top sieves and the set of sieves is kept on a mechanical shaker. Normally, 10 minutes of shaking is sufficient for most soils. The mass of soil retained on each sieve and on pan is obtained to the nearest 0.1gm

Suitability: cohesion less soils with little or no fines.

21. b. Explain wet sieve Analysis.

If the soil contains a saturated a substantial quantity of fine particles, A wet analysis required. A soil sample in the required quantity is taken, using a rifer an dried in an oven . The dried sample is taken in a tray and sacked with water. The samples stirred and lift soaking period of at least one hour.

The slurry is taken sieved through a 4.75 mm IS sieve, and washed with a jet of water . The material retained on the sieve is the gravel faction. The material retained on the 75 μ sieve is collected and dried an oven. It is then sieved through the set of the fine sieves of the size 2 mm,1 mm, 600 μ, 425 μ, 212 μ, 150 μ ,and 75 μ.

The material that would have been retained on the pan is equal to the total mass of soil minus the sum of the masses of material retain on all sieves.
22. Explain the analysis of sedimentation by pipette method.
The method is based on stokes law.

**Stokes law:**
The velocity with which a grain settles down in suspension, all other factors being equal, is dependent upon the shape, weight and size of the grains.

**Assumption:**
The coarser particles, will settle more than the finer ones.

There are 3 forces are there.

i) Drag force

ii) Weight of the sphere

iii) Buoyant force

The resisting force due to drag resistant offered by a fluid.

\[ R = 6 \pi \eta ru \]

Where,
\( \eta \) = dynamic viscosity in KN.s/m²
\( r \) = radius in m
\( u \) = velocity in m/s

Weight of the sphere = \[ \frac{4}{3} \pi r^3 \rho_s g \]

Buoyant force = \[ \frac{4}{3} \pi r^3 \rho_{sw} g \]

Equilibrium of forces in vertical direction

\[ W = U + FD \]

\[ \frac{4}{3} \pi r^3 \rho_s g = \frac{4}{3} \pi r^3 \rho_{sw} g + 6 \pi \eta ru \]

\[ V = \frac{2}{9} \frac{r^2}{\eta} (\rho_s - \rho_w) g \]
\[ D = \frac{18 \eta v}{\sqrt{(\rho_s - \rho_w)g}} \]

Now

\[ D = \frac{18 \eta v}{\sqrt{\rho_\infty g(G-1)}} \]

\[ = \frac{18 \eta v}{\sqrt{\gamma_\infty (Ss-1)}} \]

If spherical particle falls through a height He

\[ V = \frac{He}{60 t} \]

\[ \frac{He}{60 t} = \frac{1}{18} \frac{D^2}{\eta} (\rho_s - \rho_w)g \]

\[ = \frac{1}{18} \frac{D^2}{\eta} (G-1) \rho_\infty \]

\[ D = \sqrt{\frac{0.3 \eta He}{g(G-1)\rho_\infty t}} \]

\[ D = M \sqrt{\frac{He}{t}} \]

Where M is a factor, equal to \[ \sqrt{\frac{0.3 \eta}{g(G-1)\rho_\infty}} \]
23. What are the limitations of sedimentation analysis?

i) The sedimentation analysis gives the particle size in terms of equilant diameter, which is less than the particle size given by sieve analysis. The soil particles are not spherical.

ii) Stokes’ law is applicable only when the liquid is infinite. The presence of walls of the jar affects the results to same extent.

iii) In stokes law, it has been assumed that only one sphere settles, and there is no interference from other spheres. In sedimentation analysis, as many particles settled simultaneously, there is some interference.

iv) The sedimentation analysis cannot be used for particles larger than 0.2 mm as turbulent conditions develop and stokes law isn’t applicable.

24. Explain the soil classification

a) Classification based on grain size
b) Textural classification
c) AASHTO classification
d) Undefined soil classification

a) Classification based on grain size

This classification based on grain size. In this system the terms clay, silt, and gravel are used to indicate only particle size and not to signify nature of soil type.

<table>
<thead>
<tr>
<th>Clay (size)</th>
<th>Silt (size)</th>
<th>sand</th>
<th>Fine grained</th>
<th>gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Very fine</td>
<td>Fine</td>
<td>medium</td>
</tr>
<tr>
<td>0.005</td>
<td>0.05</td>
<td>0.1</td>
<td>0.25</td>
<td>0.5</td>
</tr>
</tbody>
</table>

b) Textural classification

The classification of soil exclusively based on particle size and their percentage distribution is known as textural classification. This system specifically names the soil depending on the percentage of sand, silt and clay.

c) AASHTO classification

This system is developed based on the particle size and plasticity characteristic of soil mass. A soil is classified by proceeding from left to right on the chart to find first the group into which the soil test data will fall. Soil having fine fraction are further classified based on their group index

Group index = (F-35) [0.2 + 0.005(L.L – 40)] + 0.01(F-15) (P.I-10)

F - Percentage passing 0.075 mm size
LL - Liquid Limit
P.I – Plasticity Limit
d) Unified soil classification system

This system is based on both grain size and plasticity characteristic of soil. IS system divides soils into three major groups: coarse grained and organic soils and other miscellaneous soil materials. Coarse grained soils are those with more than 50% of the material larger than 0.075 mm size. Coarse grained soils are further divided into gravels, sands. Fine grained soils are those for which more than 50% of soil finer than 0.05mm sieve size.

They divided into three subdivisions as silt, clay and organic salts and clays. Based on their plasticity nature, they are added with L, M, N, and H symbol to indicate low plastic, medium plastic, and high plastic respectively.

25. Explain the BIS classification for soil system

Indian standard classification (ISC) system adopted by Bureau of Indian Standards is in many aspects.

Soils are divided into three broad divisions

(i) Coarse-grained soils, when 50% or more of the total material by weight is retained on 75 μ IS Sieve

(ii) Fine-grained soils, when more than 50% of the total material passes 75μ IS sieve

(iii) If the soil is highly organic and contains a large percentage of organic matter and particles of decomposed vegetation, it is kept in a separate category marked as peat.


Coarse – grained soils are subdivided into gravel and sand. The soil is termed gravel (G) where more than 50% Coarse fraction (plus 75 μ) is retained on 4.75mm IS sieve, and termed sand (s) if more than 50% of the coarse friction is smaller than 4.75 mm IS sieve.

2. Fine Grained

Fine – grained soils are further divided into three subdivisions, depending upon the values of the liquid limit.

   a) Silts and clays of low compressibility – liquid limit less than 35
      (Represented by symbol H)
   b) Silts and clays of medium compressibility - these soils have liquid limit greater than 35 but less than 50.
   c) Silts and clays of high compressibility - these soils have liquid limit greater than 50 (Represented by symbol H).
### 26. Different between consolidation and compaction

<table>
<thead>
<tr>
<th>S.NO</th>
<th>CONSOLIDATION</th>
<th>COMPACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>It is a gradual process of reduction of volume under sustained, static loading.</td>
<td>It is a rapid of reduction of volume mechanical mean such as rolling, tamping, vibration.</td>
</tr>
<tr>
<td>2</td>
<td>It causes a reduction in volume of a saturated soil due to squeezing out of water from the soil.</td>
<td>In compaction, the volume of partially saturated soil decreases of air the voids at the unaltered water content.</td>
</tr>
<tr>
<td>3</td>
<td>Is a process which in nature when saturated soil deposits are subjected to static loads caused by the weight of the building</td>
<td>Is an artificial process which is done to increase the density of the soil to improve its properties before it is put to any use.</td>
</tr>
</tbody>
</table>

### 27. What are the factors affecting compaction? Explain in brief?

**i) Water content**
- a) At lower water content, the soil is stiff and others more resistance to compaction.
- b) As water content is increases, the soil particles get lubricated.
- c) Dry density of the soil increases with increases in the water content till the optimum water content is reached.
- d) After the optimum water content is reached, it becomes more difficult to force air out and to further reduce the air voids.

**ii) Amount of compaction**
At water content less than the optimum, the effect of increased compaction is more predominant. At water content more than optimum, the volume of air voids becomes almost constant and the effect of increased compaction is of significant.

**iii) Type of soil**
In general, coarse – grained soils can be compacted to higher dry density than fine grained soils. With the addition of even a small quantity of fines to a coarse-grained soil, the soil attains a much higher dry density for the same compactive effort.
Cohesive soils have air voids. Heavy clays of very high plasticity have very low dry density and very high optimum water content

**iv) Method of compaction**
The dry density achieved depends not only upon the amount of compactive effort; the dry density will depend upon whether the method of compaction utilizes kneading action, dynamic or static action.

**v) Admixture**
The compaction characteristic of the soils is improved by adding other materials known as admixtures. Ex; lime, cement and bitumen.
28. What are the different methods of compaction adopted in the field?

i) Tampers.
A hand operated tamper consists of block iron, about 3 to 5 Kg of mass, attached to a wooden rod. The tamper is lifted for about 0.30m and dropped on the soil to be compressed. Mechanical Tampers operated by compressed air or gasoline power.

ii) Rollers
a) smooth – wheel rollers
b) pneumatic – tired rollers
c) Sheep-foot rollers.

a) smooth – wheel rollers
Smooth – wheel rollers are useful finishing operations after compaction of fillers and for compacting granular base causes of highways.

b) Pneumatic – tired rollers
Pneumatic – tyred rollers use compressed air to develop the required inflation pressure. The roller compactive the soil primarily by kneading action. These rollers are effecting for compacting cohesive as well as cohesion less soils.

c) Sheep – foot rollers
The sheep – foot roller consists of a hollow drum with a large number of small projections (known as feet) on its surface. The drums are mounted on a steel frame. The drum can fill with water or ballast increases the mass. The contact pressure is generally between 700 to 4200 KN/m².

problem:29

One cubic metre of wet soil weighs 19.80 kN. If the specific gravity of soil particles is 2.70 and water content is 11%, find the void ratio, dry density and degree of saturation.


Bulk unit weight, \( \gamma_b = 19.80 \text{ kN/m}^3 \)

Water content, \( w = 11\% = 0.11 \)

Dry unit weight, \( \gamma_d = \frac{\gamma}{(1 + w)} = \frac{19.80}{(1 + 0.11)} \text{ kN/m}^3 = 17.84 \text{ kN/m}^3 \)

Specific gravity of soil particles \( G = 2.70 \)

\[ \gamma_d = \frac{G \cdot \gamma_w}{1 + e} \]

Unit weight of water, \( \gamma_w = 9.81 \text{ kN/m}^3 \)

\[ 17.84 = \frac{2.70 \times 9.81}{(1 + e)} \]

\[ (1 + e) = \frac{2.70 \times 9.81}{17.84} = 1.485 \]

Void ratio, \( e = 0.485 \)

Degree of Saturation, \( S = wG/e \)

\[ S = \frac{0.11 \times 2.70}{0.485} = 0.6124 \]

\( \therefore \) Degree of Saturation = 61.24%.
Example 30 Determine the (i) Water content, (ii) Dry density, (iii) Bulk density, (iv) Void ratio and (v) Degree of saturation from the following data:

Sample size 3.81 cm dia. × 7.62 cm h.t.
Wet weight $= 1.668$ N
Oven-dry weight $= 1.400$ N
Specific gravity $= 2.7$  
Wet weight, $W = 1.668$ N
Oven-dry weight, $W_d = 1.400$ N

Water content, $w = \frac{(1.668 - 1.400)}{1.40} \times 100\% = 19.14\%$

Total volume of soil sample, $V = \frac{\pi}{4} \times (3.81)^2 \times 7.62$ cm$^3$

Bulk unit weight, $\gamma = \frac{W}{V} = \frac{1668}{86.87} = 0.0192$ N/cm$^3$

Bulk unit weight, $= 18.84$ kN/m$^3$

Dry unit weight, $\gamma_d = \frac{\gamma}{(1 + w)} = \frac{18.84}{(1 + 0.1914)} = 15.81$ kN/m$^3$

Specific gravity of solids, $G = 2.70$

$\gamma_d = \frac{G \cdot \gamma_w}{(1 + e)}$

$\gamma_w = 9.81$ kN/m$^3$

$15.81 = \frac{2.7 \times 9.81}{(1 + e)}$

$15.81 = \frac{2.7 \times 9.81}{(1 + e)}$

$15.81 = 1.675$

$\therefore$ Void ratio, $e = 0.675$

Degree of saturation, $S = \frac{wG}{e} = \frac{0.1914 \times 2.70}{0.675} = 0.7656 = 76.56\%$.

Example 31 A soil has bulk density of 20.1 kN/m$^3$ and water content of 15%. Calculate the water content if the soil partially dries to a density of 19.4 kN/m$^3$ and the void ratio remains unchanged.  

Bulk unit weight, $\gamma = 20.1$ kN/m$^3$

Water content, $w = 15\%$

Dry unit weight, $\gamma_d = \frac{\gamma}{(1 + w)} = \frac{20.1}{(1 + 0.15)} = 17.5$ kN/m$^3$

But

$\gamma_d = \frac{G \cdot \gamma_w}{(1 + e)}$

if the void ratio remains unchanged while drying takes place, the dry unit weight also remains unchanged since $G$ and $\gamma_w$ do not change.

New value of $\gamma = 19.4$ kN/m$^3$
\[
\gamma_d = \frac{\gamma}{1 + w}
\]
\[
: \quad \gamma = \gamma_d (1 + w)
\]
or
\[
19.4 = 17.5 \quad (1 + w)
\]
\[
(1 + w) = \frac{19.4}{17.5} = 1.1086
\]
\[
w = 0.1086
\]

Hence the water content after partial drying = 10.86%.

**Example 32** The porosity of a soil sample is 35% and the specific gravity of its particles is 2.7. Calculate its void ratio, dry density, saturated density and submerged density.


Porosity, \( n = 35\% \)

Void ratio, \( e = n/(1 - n) = 0.35/0.65 = 0.54 \)

Specific gravity of soil particles = 2.7

Dry unit weight, \( \gamma_d = \frac{G \cdot \gamma_w}{(1 + e)} \)
\[
= \frac{2.7 \times 9.81}{1.54} \text{ kN/m}^3 = 17.20 \text{ kN/m}^3
\]

Saturated unit weight, \( \gamma_{sat} = \frac{(G + e)}{(1 + e)} \cdot \gamma_w \)
\[
= \frac{(2.70 + 0.54)}{1.54} \times 9.81 \text{ kN/m}^3
\]
\[
= 20.64 \text{ kN/m}^3
\]

Submerged unit weight, \( \gamma' = \gamma_{sat} - \gamma_w \)
\[
= (20.64 - 9.81) \text{ kN/m}^3
\]
\[
= 10.83 \text{ kN/m}^3.
\]

**Example 33** (i) A dry soil has a void ratio of 0.65 and its grain specific gravity is = 2.80. What is its unit weight?

(ii) Water is added to the sample so that its degree of saturation is 60% without any change in void ratio. Determine the water content and unit weight.

(iii) The sample is next placed below water. Determine the true unit weight (not considering buoyancy) if the degree of saturation is 95% and 100% respectively.

(i) Dry Soil

Void ratio,
\[ e = 0.65 \]

Grain specific gravity, \( G = 2.80 \)

Unit weight,
\[ \gamma_d = \frac{G \cdot \gamma_w}{1 + e} = \frac{2.80 \times 9.8}{1.65} \text{ kN/m}^3 = 16.65 \text{ kN/m}^3. \]

(ii) Partial Saturation of the Soil

Degree of saturation, \( S = 60\% \)

Since the void ratio remained unchanged, \( e = 0.65 \)

Water content,
\[ w = \frac{S \cdot e}{G} = \frac{0.60 \times 0.65}{2.80} = 0.1393 \]
\[ = 18.98\%. \]

Unit weight,
\[ = \frac{(G + S e)}{(1 + e)} \cdot \gamma_w = \frac{(2.80 + 0.60 \times 0.65)}{165} 9.81 \text{ kN/m}^3 \]
\[ = 18.97 \text{ kN/m}^3. \]

(iii) Sample below Water

High degree of saturation \( S = 95\% \)

Unit weight,
\[ = \frac{(G + S e)}{(1 + e)} \cdot \gamma_w = \frac{(2.80 + 0.95 \times 0.65)}{165} 9.81 \text{ kN/m}^3 \]
\[ = 20.82 \text{ kN/m}^3. \]

Full saturation, \( S = 100\% \)

Unit weight,
\[ = \frac{(G + e)}{(1 + e)} \cdot \gamma_w = \frac{(2.80 + 0.65)}{165} 9.81 \text{ kN/m}^3 \]
\[ = 20.51 \text{ kN/m}^3. \]

**Example 34** A sample of saturated soil has a water content of 35%. The specific gravity of solids is 2.65. Determine its void ratio, porosity, saturated unit weight and dry unit weight.


**Saturated soil**

Water content, \( w = 35\% \)

Specific gravity of solids, \( G = 2.65 \)

Void ratio, \( e = w/G \), in this case.
\[ e = 0.35 \times 2.65 = 0.93 \]

\[ \therefore \]

Porosity,
\[ n = \frac{e}{1 + e} = \frac{0.93}{1.93} = 0.482 = 48.20\% \]

Saturated unit weight,
\[ \gamma_{sw} = \frac{(G + e)}{(1 + e)} \cdot \gamma_w \]
\[ = \frac{(2.65 + 0.93)}{(1 + 0.93)} \times 9.81 \]
\[ = 18.15 \text{ kN/m}^3. \]
Dry unit weight,
\[ \gamma_d = \frac{G \cdot \gamma_w}{1 + e} \]
\[ = \frac{2.65 \times 9.81}{1.93} \]
\[ = 13.44 \text{ kN/m}^3. \]

**Example 35** A saturated clay has a water content of 39.3% and a bulk specific gravity of 1.84. Determine the void ratio and specific gravity of particles.


**Saturated clay**

Water content, \( w = 39.3\% \)

Bulk specific gravity, \( G_m = 1.84 \)

Bulk unit weight,
\[ \gamma = G_m \cdot \gamma_w \]
\[ = 1.84 \times 9.81 = 18.05 \text{ kN/m}^3 \]

In this case,
\[ \gamma_{sat} = 18.05 \text{ kN/m}^3 \]

\[ \gamma_{sat} = \frac{(G + e) \cdot \gamma_w}{(1 + e) \cdot \gamma_w} \]

For a saturated soil,
\[ e = w G \]

or
\[ e = 0.393 \ G \]

\[ \therefore \quad 18.05 = \frac{(G + 0.393 \ G)}{(1 + 0.393 \ G)}. \]

whence \( G = 2.74 \)

Specific gravity of soil particles = 2.74

Void ratio = 0.393 x 2.74 = 1.08.

**Example 36** The mass specific gravity of a fully saturated specimen of clay having a water content of 30.5% is 1.96. On oven drying, the mass specific gravity drops to 1.60. Calculate the specific gravity of clay.


**Saturated clay**

Water content, \( w = 30.5\% \)

Mass specific gravity, \( G_m = 1.96 \)

\[ \therefore \quad \gamma_{sat} = G_m \cdot \gamma_w = 1.96 \gamma_w \]

On oven-drying, \( G_m = 1.60 \)

\[ \therefore \quad \gamma_d = G_m \cdot \gamma_w = 1.60 \gamma_w \]

\[ \gamma_{sat} = 1.96 \gamma_w = \frac{(G + e) \gamma_w}{(1 + e)} \quad ...(i) \]

\[ \gamma_d = 1.60 \gamma_w = \frac{G \cdot \gamma_w}{(1 + e)} \quad ...(ii) \]
For a saturated soil,
\[ e = \pi G \]
\[ e = 0.305G \]

From (i),
\[ 1.96 = \frac{(G + 0.305G)}{(1 + 0.305G)} = \frac{1.305G}{(1 + 0.305G)} \]
\[ \Rightarrow \quad 1.96 + 0.598G = 1.305G \]
\[ \Rightarrow \quad G = \frac{1.960}{0.707} = 2.77 \]

From (ii),
\[ 1.60 = G/(1 + e) \]
\[ \Rightarrow \quad G = (1 + 0.305G) \times 1.6 \]
\[ \Rightarrow \quad G = 1.6 + 0.485G \]
\[ \Rightarrow \quad 0.512G = 1.6 \]
\[ \Rightarrow \quad G = 1.6/0.512 = 3.128 \]

The latter part should not have been given (additional and inconsistent data).

**Example 37** A sample of clay taken from a natural stratum was found to be partially saturated and when tested in the laboratory gave the following results. Compute the degree of saturation. Specific gravity of soil particles = 2.6; wet weight of sample = 2.50 N; dry weight of sample = 2.10 N; and volume of sample = 150 cm³. (S.V.U.—B.E.(R.R.)—Nov., 1974)

Specific gravity of soil particles, \( G = 2.60 \)
Wet weight, \( W = 2.50 \) N;
Volume, \( V = 150 \) cm³
Dry weight, \( W_d = 2.10 \) N

Water content,
\[ \omega = \frac{(W - W_d)}{W_d} \times 100 = \frac{(2.5 - 2.1)}{2.1} \times 100\% \]
\[ = \frac{0.40}{2.10} \times 100\% = 19.05\% \]

Bulk unit weight,
\[ \gamma = \frac{W}{V} = \frac{2.50}{150} = 0.0167 \) N/cm³
\[ = 16.38 \) kN/m³

Dry unit weight,
\[ \gamma_d = \frac{\gamma}{(1 + \omega)} = \frac{16.38}{(1 + 0.1905)} \) kN/m³
\[ = 13.76 \) kN/m³
Also, \( \gamma_d = \frac{W_d}{V} = \frac{2.10}{150} = 0.014 \text{ N/cm}^3 = 13.734 \text{ kN/m}^3 \)

But

\[
\gamma_d = \frac{G \gamma_w}{(1 + e)}
\]

\[
13.76 = \frac{2.6 \times 9.81}{(1 + e)}
\]

\[
(1 + e) = \frac{2.6 \times 9.81}{13.76} = 1.854
\]

\[
e = 0.854
\]

Degree of saturation,

\[
S = \frac{wG}{e} = \frac{0.1905 \times 2.6}{0.854} = 58\%
\]

**Aliter.** From the phase diagram (Fig. 2.6)

\[
V = 150 \text{ cc}
\]

\[
W = 2.50 \text{ N}
\]

\[
V = 150 \text{ cc}
\]

\[
W = 2.50 \text{ N}
\]

\[
W_d = W_s = 2.10 \text{ N}
\]

\[
V_V = 69.23 \text{ cm}^3
\]

\[
V_W = 40 \text{ cm}^3
\]

\[
V_S = 80.77 \text{ cm}^3
\]

\[
W_w = 0.40 \text{ N}
\]

\[
W_s = 2.10 \text{ N}
\]

\[
W = 2.50 \text{ N}
\]

**Fig. 2.6** Phase diagram (Example 2.9)

\[
W_w = (2.50 - 2.10) \text{ N}
\]

\[
= 0.40 \text{ N}
\]

\[
V_w = \frac{W_w}{\gamma_w} = \frac{0.40}{0.01} = 40 \text{ cm}^3
\]
\[ V_s = \frac{W_s}{\gamma_s} - \frac{W_z}{G_s \gamma_w} = \frac{2.10}{2.6 \times 0.01} = 80.77 \text{ cm}^3 \]

\[ V_v = (V - V_s) = (150 - 80.77) = 69.23 \text{ cm}^3 \]

Degree of saturation, \[ S = \frac{V_w}{V_v} \]

\[ S = \frac{40}{69.23} = 0.578 \]

\[ S = \frac{40}{69.23} = 0.578 \]

\[ \therefore \text{ Degree of saturation} = 57.8\% \]

Thus, it may be observed that it may sometimes be simpler to solve numerical problems by the use of the soil-phase diagram.

**Note.** All the illustrative examples may be solved with the aid of the soil-phase diagram or the unit-phase diagram also; however, this may not always be simple.

**Example 38** In a specific gravity test with pyknometer, the following observed readings are available:

- Weight of the empty pyknometer = 7.50 N
- Weight of pyknometer + dry soil = 17.30 N
- Weight of pyknometer + dry soil + water filling the remaining volume = 22.45 N
- Weight of pyknometer + water = 16.30 N

Determine the specific gravity of the soil solids, ignoring the effect of temperature.

The given weights are designated \( W_1 \) to \( W_4 \) respectively.

Then,

- the weight of dry soil solids, \( W_s = W_2 - W_1 \)
  \[ = (17.30 - 7.50) \text{ N} = 9.80 \text{ N} \]

The specific gravity of soil solids is given by Eq. 3.1:

\[ G = \frac{W_s}{W_s - (W_3 - W_4)} \] (ignoring the effect of temperature)
\[
= \frac{9.80}{9.80 - (22.45 - 16.30)}
\]
\[
= \frac{9.80}{9.80 - 16.15} = 2.685
\]

\[
\therefore \text{Specific gravity of the soil solids} = 2.685.
\]

**Example 39** In a specific gravity test, the weight of the dry soil taken is 0.66 N. The weight of the pyknometer filled with this soil and water is 6.756 N. The weight of the pyknometer full of water is 6.3395 N. The temperature of the test is 30°C. Determine the grain specific gravity, taking the specific gravity of water at 30°C as 0.99568.

Applying the necessary temperature correction, report the value of \( G \) which would be obtained if the test were conducted at 4°C and also at 27°C. The specific gravity values of water at 4°C and 27°C are respectively 1 and 0.99654.

- Weight of dry soil taken, \( W_2 = 0.66 \) N
- Weight of pyknometer + soil + water \( (W_3) = 6.756 \) N
- Weight of pyknometer + Water \( (W_4) = 6.3395 \) N
- Temperature of the Test \( (T) = 30^\circ \text{C} \)
- Specific gravity of water at 30°C \( (G_{w_2}) = 0.99568 \)

By Eq. 3.4,

\[
G = \frac{W_2 \cdot G_{w_2}}{W_2 - (W_3 - W_4)}
\]

\[
= \frac{0.66 \times 0.99568}{0.66 - (6.756 - 6.3395)} = 2.69876 \approx 2.70
\]

If the test were conducted at 4°C, \( G_{w_2} = 1 \)

Specific gravity of kerosene at 27°C = 0.773

Determine the specific gravity of the soil solids.

What will be the value if it has to be reported at 4°C?

Assume the specific gravity of water at 27°C as 0.99654.

Let the weight be designated as \( W_1 \) through \( W_4 \) in that order.

Wt of dry clay sample, \( W_2 = (W_2 - W_1) = (0.816 - 0.6025) \) N = 0.2135 N

By Eq. 3.2,
\[ G = \frac{W_s \cdot G_k}{W_s - (W_3 - W_4)} \]

\( G_k \) here is given as 0.773.

\[ \therefore \quad G = \frac{0.2135 \times 0.773}{0.2135 - (2.5734 - 2.4217)} = 2.67 \]

If the value has to be reported at 4°C, by Eq. 3.3,

\[ G_{T_2} = G_{T_1} \cdot \frac{G_{w_{T_2}}}{G_{w_{T_1}}} \]

\[ \therefore \quad G_{4°C} = G_{27°C} \cdot \frac{1}{0.99654} = \frac{2.67 \times 1}{0.99654} = 2.68. \]

**Example 40** In a specific gravity test, the following observation were made:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of dry soil</td>
<td>1.04 N</td>
</tr>
<tr>
<td>Weight of bottle + soil + water</td>
<td>5.38 N</td>
</tr>
<tr>
<td>Weight of bottle + water</td>
<td>4.756 N</td>
</tr>
</tbody>
</table>

What is the specific gravity of soil solids. If, while obtaining the weight 5.38 N, 3 ml of air remained entrapped in the suspension, will the computed value of \( G \) be higher or lower than the correct value? Determine also the percentage error.

Neglect temperature effects.

Neglecting temperature effects, by Eq. 3.1,

\[ G = \frac{W_s}{W_s - (W_3 - W_4)} \]

In this case, \( W_s = 1.04 \) N; \( W_3 = 5.38 \) N; \( W_4 = 6.756 \) N

\[ \therefore \quad G = \frac{1.04}{1.04 - (5.38 - 4.756)} = 2.50 \]

If some air is entrapped while the weight \( W_3 \) is taken, the observed value of \( W_3 \) will be lower than if water occupied this air space. Since \( W_3 \) occurs with a negative sign in Eq. 3.1 in the denominator, the computed value of \( G \) would be lower than the correct value.

Since the air entrapped is given as 3 ml, this space, if occupied by water, would have enhanced the weight \( W_3 \) by 0.03 N.

\[ \therefore \quad \text{Correct value of } G = \frac{1.04}{1.04 - (5.41 - 4.756)} = \frac{1.040}{0.386} = 2.694 \]
Percentage error: \[ \frac{(2.694 - 2.500)}{2.694} \times 100 = 7.2. \]

**Example 41** A pyknometer was used to determine the water content of a sandy soil. The following observation were obtained:

- Weight of empty pyknometer = 8 N
- Weight of pyknometer + wet soil sample = 11.60 N
- Weight of pyknometer + wet soil + water filling remaining volume = 20 N
- Weight of pyknometer + water = 18 N
- Specific gravity of soil solids (determined by a separate test) = 2.66

Compute the water content of the soil sample.

The weights may be designated \( W_1 \) through \( W_4 \) in that order.

By Eq. 3.6,

\[ w = \left[ \frac{(W_2 - W_1)}{(W_3 - W_4)} \left( \frac{G - 1}{G} \right) - 1 \right] \times 100 \]

Substituting the values,

\[ w = \left[ \frac{(11.60 - 8.00)}{(20 - 18)} \times \frac{(2.66 - 1)}{2.66} - 1 \right] \times 100 \]

\[ = \left[ \frac{3.6}{2.0} \times \frac{1.66}{2.66} - 1 \right] \times 100 \]

\[ = (1.1233 - 1) \times 100 = 12.33 \%
\]

**Example 3.6:** A soil sample with a grain specific gravity of 2.67 was filled in a 1000 ml container in the loosest possible state and the dry weight of the sample was found to be 14.75 N. It was then filled at the densest state obtainable and the weight was found to be 17.70 N. The void ratio of the soil in the natural state was 0.63. Determine the density index in the natural state.

\[ G = 2.67 \]

**Loosest state:**

- Weight of soil = 14.75 N
- Volume of solids = \( \frac{14.75}{0.0267} \) cm\(^3\) = 552.4 cm\(^3\)

**Volume of voids** = (1000 - 552.4) cm\(^3\) = 447.6 cm\(^3\)
Void ratio, $e_{\text{max}} = \frac{447.6}{552.4} = 0.810$

Densest state:

Weight of soil = 17.70 N

Volume of solids = \frac{17.70}{0.0257} \text{ cm}^3 = 662.9 \text{ cm}^2

Void ratio, $e_{\text{min}} = \frac{337.1}{662.9} = 0.508$

Void ratio in the natural state, $e = 0.63$

Density Index, $I_D = \frac{(e_{\text{max}} - e)}{(e_{\text{max}} - e_{\text{min}})}$, by (Eq. 3.8)

$$= \frac{0.810 - 0.630}{0.810 - 0.508} = \frac{0.180}{0.302} = 0.596$$

$\therefore I_D = 59.6\%$.

**Example 42** The dry unit weight of a sand sample in the loosest state is 13.34 kN/m$^3$ and in the densest state, it is 21.19 kN/m$^3$. Determine the density index of this sand when it has a porosity of 33%. Assume the grain specific gravity as 2.68.

$\gamma_{\text{min}}$ (loosest state) = 13.34 kN/m$^3$

$\gamma_{\text{max}}$ (densest state) = 21.19 kN/m$^3$

Porosity, $n = 33\%$

Void ratio, $e_0 = \frac{n}{(1 - n)} = \frac{33}{67} = 0.49$

$\gamma_0 = \frac{G \cdot \gamma_w}{(1 - e_0)} = \frac{2.68 \times 9.81}{1 + 0.49} \text{ kN/m}^3 = 17.64 \text{ kN/m}^3$

Density Index, $I_D$ (by Eq. 3.10)

$$= \frac{(\gamma_{\text{max}})}{(\gamma_0)} \left( \frac{\gamma_0 - \gamma_{\text{min}}}{\gamma_{\text{max}} - \gamma_{\text{min}}} \right)$$

$$= \frac{21.19}{17.64} \times \frac{(17.64 - 13.34)}{(21.19 - 13.34)} = \frac{21.19}{17.64} \times \frac{4.30}{7.85} = 0.658 = 65.8\%$$

Alternatively:

$$\gamma_{\text{min}} = \frac{G \cdot \gamma_w}{(1 - e_{\text{max}})} = \frac{2.68 \times 9.81}{1 + e_{\text{max}}}$$
\[ e_{\min} = 0.241 \]
\[ I_D = \frac{(c_{\max} - c_0)}{(c_{\max} - c_{\min})}, \text{ (by Eq. 3.8)} \]
\[ = \frac{(0.971 - 0.49)}{(0.971 - 0.241)} = 0.48 \]
\[ = \frac{0.73}{0.73} = 56.8\% \]

**Example 43** The following data were obtained during an *in-situ* unit wt determination of an embankment by the sand-replacement method:

- Volume of calibrating can = 1000 ml
- Weight of empty can = 9 N
- Weight of can + sand = 25 N
- Weight of sand filling the conical portion of the sand-pouring cylinder = 4.5 N

**Initial weight of sand-pouring cylinder + sand = 54 N**

**Weight of cylinder + sand, after filling the excavated hole = 41.4 N**

- Wet weight of excavated soil = 9.36 N
- *In-situ* water content = 9%

Determine the *in-situ* unit weight and *in-situ* dry unit weight.

**Sand-replacement method of *in-situ* unit weight determination:**

- Weight of sand filling the calibrating can of volume 1000 ml = (25 - 9) N = 16 N
- Unit weight of sand = 16/1000 N/cm³ = 0.016 N/cm³
- Weight of sand filling the excavated hole
  - and conical portion of the sand pouring cylinder = (54 - 41.4) = 12.60 N
  - Weight of sand filling the excavated hole = (12.6 - 4.5) = 8.10 N

\[ \text{Volume of the excavated hole} = \frac{8.10}{0.016} \text{ cm}^3 = 506.25 \text{ cm}^3 \]

- Weight of excavated soil = 9.36 N
- *In-situ* unit weight, \( \gamma \) = 9.36/506.25 N/cm³ = 18.15 kN/m³
- Water content, \( w \) = 9%

\[ In-situ \text{ dry unit weight, } \gamma_d = \frac{\gamma}{(1 + w)} = \frac{18.15}{(1 + 0.09)} \text{ kN/m}^3 = 16.67 \text{ kN/m}^3. \]

**Example 44** A field density test was conducted by core-cutter method and the following data was obtained:

- Weight of empty core-cutter = 22.80 N
- Weight of soil and core-cutter = 50.05 N
- Inside diameter of the core-cutter = 90.0 mm
- Height of core-cutter = 180.0 mm
Height of core-cutter = 180.0 mm
Weight or wet sample for moisture determination = 0.5405 N
Weight of oven-dry sample = 0.5112 N
Specific gravity of soil grains = 2.72
Determine (a) dry density, (b) void-ratio, and (c) degree of saturation.

(S.V.U.—B.E.(Part-time)—FE—April, 1982)

Weight of soil in the core-cutter \( W = (50.05 - 22.80) = 27.25 \) N
Volume of core-cutter \( V = \frac{27.25}{1145.11} \) cm³ = 23.34 kN/m³
Wet unit weight of soil \( \gamma = \frac{W}{V} = \frac{27.25}{1145.11} \) N/cm³
Weight of oven-dry sample = 0.5112 N
Weight of moisture = \( 0.5405 - 0.5112 = 0.0293 \) N
Moisture content, \( w = \frac{0.0293}{0.5112} \times 100\% = 5.73\% \)

Dry unit weight, \( \gamma_d = \frac{\gamma}{(1+w)} = \frac{23.34}{(1 + 0.0573)} \) kN/m³

Grain specific gravity, \( G = 2.72 \)

\[ \gamma_d = \frac{G \cdot \gamma_w}{(1 + e)} \text{ or } 22.075 = \frac{2.72 \times 9.81}{(1 + e)} \]

whence, the void-ratio, \( e = 0.21 \)

Degree of saturation, \( S = \frac{wG}{e} = \frac{0.0573 \times 2.72}{0.21} = 74.2\% \).

**Example 45** A soil sample consists of particles ranging in size from 0.6 mm to 0.02 mm. The average specific gravity of the particles is 2.66. Determine the time of settlement of the coarsest and finest of these particles through a depth of 1 metre. Assume the viscosity of water as 0.001 N·sec/m² and the unit weight as 9.8 kN/m³.

By Stokes’ law (Eq. 3.19),

\[ v = \frac{(1/180) \cdot (\gamma_w/\mu_w) (G - 1)D^2}{(1 + e)} \]

where \( v = \) terminal velocity in cm/sec,

\[ \gamma_w = \text{unit weight of water in kN/m}^3, \]

\[ \mu_w = \text{viscosity of water in N·sec/m}^2, \]
\[ G = \text{grain specific gravity, and} \]
\[ D = \text{size of particle in mm.} \]

\[ \therefore \ v = \frac{1}{180} \times \frac{9.80}{0.001} \ (2.66 - 1)D^2 = 90D^2 \]

For the coarsest particle, \( D = 0.6 \text{ mm} \)

\[ v = 90 \times (0.6)^2 \text{ cm/sec.} = 32.4 \text{ cm/sec.} \]
\[ t = \frac{h}{v} = \frac{100.0}{32.4} \text{ sec.} = 3.086 \text{ sec.} \]

For the finest particle, \( D = 0.02 \text{ mm} \).

\[ v = 90(0.02)^2 \text{ cm/sec.} = 0.036 \text{ cm/sec.} \]
\[ t = \frac{h}{v} = \frac{100.000}{0.036} \text{ sec.} = 2777.78 \text{ sec.} = 46 \text{ min.} 17.78 \text{ sec.} \]

This time of settlement of the coarsest and finest particles through one metre are nearly 4 sec. and 46 min. 18 sec. respectively.

**Example 46** In a pipette analysis, 0.5 N of dry soil (fine fraction) of specific gravity 2.72 were mixed in water to form half a litre of uniform suspension. A pipette of 10 ml capacity was used to obtain a sample from a depth of 10 cm after 10 minutes from the start of sedimentation. The weight of solids in the pipette sample was 0.0032 N. Assuming the unit weight of water and viscosity of water at the temperature of the test as 9.8 kN/m² and 0.001 N-sec/m² respectively, determine the largest size of the particles remaining at the sampling depth and percentage of particles finer than this size in the fine soil fraction taken. If the percentage of fine fraction in the original soil was 50, what is the percentage of particles finer computed above in the entire soil sample?

By Eqs. 3.21 and 3.22,

\[ D = K \sqrt{\frac{H}{t}} \]

where

\[ K = \frac{3\gamma_w}{\sqrt{\gamma_w(G - 1)}} \]

\[ \therefore \ K = \frac{3 \times 0.001}{\sqrt{9.8 \times (2.72 - 1)}} = 0.0133 \text{ mm} \sqrt{\text{min}} \sqrt{\text{cm}} \]

\[ D = 0.0133 \sqrt{\frac{10}{10}} \text{ mm} = 0.0133 \text{ mm} \]

This is the largest size remaining at the sampling depth.

By Eq. 3.26,

\[ N_r = \left( \frac{W_p}{W_s} \right) \left( \frac{V}{V_p} \right) \times 100 \]
Here \( W_p = 0.0032 \text{ N} \); \( W_s = 0.5 \text{ N} \); \( V = 500 \text{ ml} \); \( V_p = 10 \text{ ml} \).

\[
N_f = \frac{0.0032}{0.50} \times \frac{500}{10} \times 100 = 32\%
\]

Thus, the percentage of particles finer than 0.0133 mm is 32.

By Eq. 3.28, \( N = N_f (W_p / W) \)

Here \( W_p / W \) is given as 0.50.

\[
N = 32 \times 0.50 = 16
\]

Hence, this percentage is 16, based on the entire sample of soil.

**Example 47**  In a hydrometer analysis, the corrected hydrometer reading in a 1000 ml uniform soil suspension at the start of sedimentation was 28. After a lapse of 30 minutes, the corrected hydrometer reading was 12 and the corresponding effective depth 10.5 cm. The specific gravity of the solids was 2.70. Assuming the viscosity and unit weight of water at the temperature of the test as 0.001 N·s/m² and 9.8 kN/m³ respectively, determine the weight of solids mixed in the suspension, the effective diameter corresponding to the 30-min. reading and the percentage of particles finer than this size.

The corrected hydrometer reading initially, \( R_{hi} = 28 \)

\[
\gamma_i = 0.01028 \text{ N/cm}^3
\]

But, by Eq. 3.29,

\[
\gamma_i = \gamma_w + \frac{[G - 1]}{G} \frac{W}{V}
\]

Substituting,

\[
0.01028 = 0.01 + \frac{(2.7 - 1)}{2.7} \times \frac{W}{1000}
\]

whence \( W = \frac{0.028 \times 2.7}{17} \text{ N} = 0.445 \text{ N} \)

\[
\therefore \text{The weight of solid mixed in the suspension} = 0.445 \text{ N}.
\]

By Eqs. 3.21 and 3.22,

\[
D = K \sqrt{\frac{H}{t}}
\]

where

\[
K = \sqrt{\frac{3 \mu_w}{\gamma_w (G - 1)}}
\]

\[
\therefore K = \sqrt{\frac{3 \times 0.001}{9.8 \times (2.7 - 1)}} = 0.01342 \text{ mm} \sqrt{\text{min}} / \sqrt{\text{cm}}
\]

\[
\therefore D = 0.01342 \sqrt{\frac{10.5}{30}} \text{ mm} = 0.00794 \text{ mm} \approx 0.008 \text{ mm}
\]
The effective diameter corresponding to the 30-min, reading = 0.008 mm

By Eq. 3.34,

\[ N = \frac{G \cdot W \cdot V \cdot R_h}{(G - 1) \cdot W \cdot 10} \]

\[ \therefore N = \frac{2.7 \times 0.01 \times 1000}{0.445 \times 10} = 42.83 \]

\[ \therefore \text{The percentage of particles finer than 0.008 mm is 48.} \]

**Example 48** The liquid limit of a clay soil is 56% and its plasticity index is 15%.

(a) In what state of consistency is this material at a water content of 45%?

(b) What is the plastic limit of the soil?

(c) The void ratio of this soil at the minimum volume reached on shrinkage, is 0.88. What is the shrinkage limit, if its grain specific gravity is 2.71?

Liquid limit, \( W_L = 56\% \)

Plasticity index, \( I_p = 15\% \)

\[ I_p = w_L - w_p \text{, by Eq. 3.37.} \]

\[ \therefore 15 = 56 - w_p \]

\[ \text{Whence the plastic limit, } w_p = (56 - 15) = 41\% \]

\[ \therefore \text{At a water content of 45\%, the soil is in the plastic state of consistency.} \]

Void ratio at minimum volume, \( e = 0.88 \)

Grain specific gravity, \( G = 2.71 \)

Since at shrinkage limit, the volume is minimum and the soil is still saturated,

\[ e = w_s G \]

\[ w_s = e/G = 0.88/2.71 = 32.5\% \]

\[ \therefore \text{Shrinkage limit of the soil = 32.5\%.} \]

**Example 49** A soil has a plastic limit of 25% and a plasticity index of 30. If the natural water content of the soil is 34%, what is the liquidity index and what is the consistency index?

How do you describe the consistency?

Plastic limit, \( w_p = 25\% \)

Plasticity index, \( I_p = 30 \)

By Eq. 3.37, \( I_p = w_L - w_p \)
:. Liquid limit, 
\[ w_L = I_p + w_p = 30 + 25 = 55\% \]
By Eq. 3.40,
\[ I_L = \frac{(w - w_p)}{I_p} \]

Liquidity index,
\[ I_L = \frac{(34 - 25)}{80} = 0.30 \]
By Eq. 3.39,
\[ I_c = \frac{(w_L - w)}{I_p} \]

Consistency index,
\[ I_c = \frac{(55 - 34)}{30} = 0.70 \]

The consistency of the soil may be described as ‘medium soft’ or ‘medium stiff’.

**Example 50** A fine grained soil is found to have a liquid limit of 90% and a plasticity index of 50. The natural water content is 28%. Determine the liquidity index and indicate the probable consistency of the natural soil.

Liquid limit, 
\[ w_L = 90\% \]
Plasticity index, 
\[ I_p = 50 \]
By Eq. 3.37, 
\[ I_p = w_L - w_p \]
:. Plastic limit, 
\[ w_p = w_L - I_p = 90 - 50 = 40\% \]
The natural water content, 
\[ w = 28\% \]
Liquidity index, \( I_L \), by Eq. 3.40, is given by
\[ I_L = \frac{w - w_p}{I_p} = \frac{28 - 40}{50} = -\frac{12}{50} = -0.24 \text{ (negative)} \]

Since the liquidity index is negative, the soil is in the semi-solid state of consistency and is stiff; this fact can be inferred directly from the observation that the natural moisture content is less than the plastic limit of the soil.

**Example 51** A clay sample has void ratio of 0.50 in the dry condition. The grain specific gravity has been determined as 2.72. What will be the shrinkage limit of this clay?

The void ratio in the dry condition also will be the void ratio of the soil even at the shrinkage limit: but the soil has to be saturated at this limit.

For a saturated soil,
\[ e = wG \]
or
\[ w = e/G \]
:. 
\[ w_L = e/G = 0.50/2.72 = 18.4\% \]

Hence the shrinkage limit for this soil is 18.4%.

**Example 52** The following are the data obtained in a shrinkage limit test:
Initial weight of saturated soil \( = 0.956 \text{ N} \)
Initial volume of the saturated soil \( = 68.5 \text{ cm}^3 \)
Final dry volume = 24.1 cm³  
Final dry weight = 0.435 N

Determine the shrinkage limit, the specific gravity of grains, the initial and final dry unit weight, bulk unit weight, and void ratio. (S.V.U.—B.Tech. (Part-time)—Sept. 1982)

From the data,
Initial water content, \[ w_i = \frac{(0.956 - 0.435)}{0.435} \times 100 = 119.77\% \]

By Eq. 3.48, the shrinkage limit is given by
\[
W_s = \left[ w_i - \frac{(V_i - V_d)}{W_d} \right] \times 100 \\
= \left[ 1.1977 - \frac{(68.5 - 24.1)}{0.435} \times 0.01 \right] \times 100 = 17.70\% \\

Final dry unit weight = \frac{0.435}{24.1} N/cm³ = 17.71 kN/m³

Initial bulk unit weight = \frac{0.956}{68.5} N/cm³ = 13.70 kN/m³

By Eq. 3.53,

Grain specific gravity = \frac{1}{\left( \frac{\gamma_w - w_i}{\gamma_d} \right)} = \frac{1}{\left( \frac{9.81 - 17.70}{17.71} \right)} = 2.63

Initial dry unit weight = \frac{\gamma_d}{(1 + w_i)} = \frac{13.70}{(1 + 1.1977)} = 6.28 kN/m³

Initial void ratio = \frac{w_i G}{1.1977} \times 2.65 = 3.17

Final void ratio = \frac{w_s G}{0.1770 \times 2.65} = 0.47.

Example 53: The Atterberg limits of a clay soil are: Liquid limit = 75%; Plastic limit = 45%; and Shrinkage limit = 25%. If a sample of this soil has a volume of 30 cm³ at the liquid limit and a volume 16.6 cm³ at the shrinkage limit, determine the specific gravity of solids, shrinkage ratio, and volumetric shrinkage.

The phase diagrams at liquid limit, shrinkage limit, and in the dry state are shown in Fig. 3.23:
Difference in the volume of water at LL and SL = \((30 - 16.6)\) \(\text{cm}^3 = 13.4\) \(\text{cm}^3\)

Corresponding difference in weight of water = 0.134 N

But this is \((0.75 - 0.25) W_d\) or 0.50 \(W_d\) from Fig. 3.23.

\[ \therefore \quad 0.50 W_d = 0.134 \]

\[ W_d = 0.268\ N \]

Weight of water at SL = 0.25 \(W_d\) = 0.25 \(\times\) 0.268 = 0.067 N

\[ \therefore \quad \text{Volume of water at SL} = 6.7 \text{ cm}^3 \]

Volume of solids, \(V_s = \text{Total volume at SL} - \text{volume of water at SL.} \]
\[ = (16.6 - 6.7) \text{ cm}^3 = 9.9 \text{ cm}^3. \]

Weight of solids, \(W_d = 0.268\ N \]

\[ \therefore \quad \gamma_s = \frac{0.268}{9.9} \text{ N/cm}^3 = 0.027 \text{ N/cm}^3 = 27 \text{ kN/m}^3 \]

\[ \therefore \quad \text{Specific gravity of solids} = \frac{\gamma_s}{\gamma_w} = \frac{27}{9.81} = 2.71 \]

Shrinkage ratio, \(R = \frac{W_d}{V_d} = \frac{26.8}{16.6} = 1.61 \)

Volumetric shrinkage, \(V_s = R(w_i - w_s) = R(w_s - w_i), \) here
\[ = 1.61 \times (75 - 25) = 80.5\% . \]

**Example 54** The mass specific gravity of a saturated specimen of clay is 1.84 when the water content is 38%. On oven drying the mass specific gravity falls to 1.70. Determine the specific gravity of solids and shrinkage limit of the clay.

For a saturated soil,

\[ e = w.G \]

\[ \therefore \quad e = 0.38 G \]

Mass specific gravity in the saturated condition

\[ = \frac{\gamma_{sat}}{\gamma_w} = \frac{(G + e)}{(1 + e)} = \frac{(G + 0.38 G)}{(1 + 0.38 G)} \]

\[ \therefore \quad 1.84 = \frac{1.38 G}{1 + 0.38 G} \]

whence

\[ G = 2.71 \]

\[ \therefore \quad \text{Specific gravity of the solids} = 2.71 \]

By Eq. 3.50, the shrinkage limit is given by

\[ w_s = \left( \frac{\gamma_w}{\gamma_d} - 1 \right) \times 100 \]

where \( \gamma_d = \text{dry unit weight in dry state.} \)

But, \( \gamma_d = (\text{mass specific gravity in the dry state})\ \gamma_w \)

\[ = 1.70 \ \gamma_w \]
Example 55  A saturated soil sample has a volume of 23 cm³ at liquid limit. The shrinkage limit and liquid limit are 13% and 45%, respectively. The specific gravity of solids is 2.73. Determine the minimum volume which can be attained by the soil.

The minimum volume which can be attained by the soil occurs at the shrinkage limit. The phase diagrams of the soil at shrinkage limit and at liquid limit are shown in Fig. 3.24:

Fig. 3.24 Phase diagrams (Example 3.20)

At liquid limit,

\[ V_{L} = 23 \text{ cm}^3 \]

\[ V_s = \frac{W_s}{G \cdot \gamma_w} = \frac{1000}{9.81 \times 10^{-3}} = \frac{9.81}{9.81} W_s \text{ cm}^3 \]

whence \( W_s = 0.2818 \text{ N} \)

At shrinkage limit,

the volume,

\[ V_m = V_s + 0.18 W_s = \left( \frac{0.2818}{0.0273} + 18 \times 0.2818 \right) \text{ cm}^3 = 15.4 \text{ cm}^3 \]

Example 56  An oven-dry soil sample of volume 225 cm³ weighs 3.90 N. If the grain specific gravity is 2.72, determine the void-ratio and shrinkage limit. What will be the water content which will fully saturate the sample and also cause an increase in volume equal to 8% of the original dry volume?

Dry unit weight of the oven-dry sample

\[ \gamma_d = \frac{3.9}{225} \text{ N/cm}^3 = 17.33 \text{ kN/m}^3 \]

But

\[ \gamma_d = \frac{G \cdot \gamma_w}{(1 + e)} \]

\[ 17.33 = \frac{2.72 \times 10}{(1 + e)} \]

\[ e = 0.57 \]

Void-ratio \( = 0.57 \)

Shrinkage limit,

\[ w_s = e/G = 0.57/2.72 = 21\% \]

The conditions at shrinkage limit and final wet state are shown in Fig. 3.25:
Fig. 3.25 Phase-diagrams of soil (Example 3.21)

\[ V_s = \frac{W_d}{G} = \frac{3.90}{0.0272} = 143.38 \text{ cm}^3 \]

Volume in the final wet state, \[ V = (225 + 0.08 \times 225) = 243 \text{ cm}^3 \]

Volume of water in the final wet state, \[ V_w = (243 - 143.38) \text{ cm}^3 = 99.62 \text{ cm}^3 \]

Weight of water in the final wet state = 0.9962 N

Water content in the final wet state = \[ \frac{0.9962}{3.90} = 25.5\% \]