UNIT II – UNIFORM FLOW

2 MARK QUESTIONS AND ANSWERS

1. Define uniform flow. Give examples.
   Uniform flow is a fluid flow in which the velocity of any given instant does not change both in magnitude and direction with respect to space. Mathematically,

   \[ \left( \frac{\partial v}{\partial s} \right) = 0 \]

   **Example:**
   - Open channel flow with constant depth of water
   - Flow through uniform diameter pipes.

2. What are the instruments used for measuring velocity in open channels?
   [May'06, May'07May'08 May'09]
   Velocity of flow is measured by various instruments such as Pitot tube, Current meter, hot wire anemometer, floats and Laser Doppler velocimetry.

3. What is cup type current meter?
   In this type, series of conical cups called revolving element are mounted on a spindle vertically at right angle to the direction of flow.

4. Give some applications of laser Doppler Anemometer.
   1. It is used for the flow between blades of a turbine.
   2. It is used in combustion and flame phenomena in gas turbines.
   3. It is used in Jet propulsion systems.
   4. It is used for measuring the blood flows.
   5. In remote sensing of wind velocities.
5. Write down the Manning’s formula for determining velocity of flow in an open channel.

\[ V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \]

Where

- \( n \) = Manning’s Roughness Co-efficient
- \( V \) = mean velocity of flow in m/s
- \( R \) = Hydraulic radius of the channel in m
- \( S \) = Channel bed Slope

6. List the factors affecting Manning’s roughness coefficient. [Nov’08] The following factors affecting Manning’s roughness coefficient are:

1. Surface roughness
2. Vegetation growth
3. Channel irregularities
4. Sitting and scouring
5. Stage (water surface elevation) and discharge
6. Transport of suspended and bed material.

7. What are the condition for obtaining most economical circular channel section for maximum velocity and discharge?

   a. **Condition for maximum velocity of circular section**
   
   (i) Depth of flow is 0.81 times the diameter of the circular channel. (ii) Hydraulic radius is equal to 0.3 times the diameter of channel. (iii) Angle subtended by water surface from the centre,

   \[ \theta = 257^0 30' \]

   b. **Condition for maximum discharge of circular section**

   (iv) Depth of flow is 0.91 times the diameter of the circular channel. (v) Hydraulic radius is equal to 0.286 times the diameter of channel. (vi) Angle subtended by water surface from the centre, \( 20 = 308^0 \).
8. Show that maximization of discharge required minimization of the wetted perimeter of the channel for a given area of flow. [May’10]
   For a given channel slope, roughness coefficient and area of flow, the maximum discharge of channel is obtained when the wetted perimeter is minimum.

   \[ \text{For Wetter perimeter } (P) \text{ to be min} \quad \frac{dP}{dy} = 0 \]

   second derivative of \( P \) is positive, the condition of minimum \( P \) is obtained

9. Define non-erodible channels.
   Channels which are constructed from materials, such as concrete, masonry and metal can withstand erosion under all including most extreme conditions are called as non-erodible sections

10. What are the factors considered while designing non-erodible channels?
   The following factors considered while designing the non-erodible channels are:
   a) Manning’s constant ‘n’ value of the material
   b) Channel slope
   c) Free board

11. How Stickler equation can be used to calculate roughness coefficient?
   Sticker formula is used to determine Manning’s constant ‘n’ in non-erodible channels

12. Define the term most economical section of the channel
   A section of the channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction depend up on the excavation and lining to keep the cost minimum The wetted perimeter for a given discharge should be minimum.

   \[ n = (0.038d)^{\frac{1}{6}} \]

   Where

   \( d = \text{Particle size diameter in meter} \)
13. What is meant by conveyance of the channel?
   The conveyance of the channel is denoted by \( k \) and is given by \( k = AC \sqrt{(mi)} \).

14. Define the term most economical section:
   A section of the channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction mainly depend up on the excavation and lining to keep the cost minimum, the wetted perimeter for a given discharge should be minimum.

15. What are the conditions of rectangular channel of best section?
   The two conditions are
   - breadth is equal to two times the depth \( (b=2d) \)
   - hydraulic mean depth is equal to half the depth \( (m=d/2) \)

16. Write down the conditions for the most economical trapezoidal channel?
   1. Half the top width is equal to one of sloping side \( (b+2 nd)/2 = d \sqrt{(1+n^2)} \)
   2. Hydraulic mean depth is equal to half the depth.

17. Write down the conditions of most economical circular channel with maximum velocity?
   \( \theta = 128^\circ 45' \)
   \( d = 0.81D \)
   \( m = 0.3D \)

18. What is the best side slope for trapezoidal channel?
   \( \theta = 60^\circ \) is the best side slope for trapezoidal channel.

19. What is meant by wetted perimeter?
   The wetted perimeter \( (p) \) is the length of the line of intersection of the channel wetted surface with the cross section plan normal to the direction of flow.

20. Uniform flow
   If the flow velocity at a given instant of time does not vary within a given length of channel, then the flow is called uniform flow.
21. Manning’s formula

The Manning formula, known also as the Gauckler–Manning formula, or Gauckler–Manning–Strickler formula in Europe, is an empirical formula for open channel flow, or free-surface flow driven by gravity. It was first presented by the French engineer Philippe Gauckler in 1867 and later re-developed by the Irish engineer Robert Manning in 1890.

22. Hydraulic radius

The hydraulic radius is a measure of a channel flow efficiency. Flow speed along the channel depends on its cross-sectional shape (among other factors), and the hydraulic radius is a characterization of the channel that intends to capture such efficiency. Based on the ‘constant shear stress at the boundary’ assumption, hydraulic radius is defined as the ratio of the channel's cross-sectional area of the flow to its wetted perimeter (the portion of the cross-section's perimeter that is wet).

23. Darcy Weisbach equation

In fluid dynamics, the Darcy Weisbach equation is a phenomenological equation, which relates the head loss or pressure loss due to friction along a given length of pipe to the average velocity of the fluid flow. The equation is named after Henry Darcy and Julius Weisbach.

24. Normal depth

Normal depth is the depth of flow in a channel or culvert when the slope of the water surface and channel bottom is the same and the water depth remains constant. Normal depth occurs when gravitational force of the water is equal to the friction drag along the culvert and there is no acceleration of flow. In culverts, water flows at normal depth when outside the influence of the inlet and outlet tail water. Normal depth is undefined for culverts placed at horizontal or adverse slopes.
16 MARK QUESTIONS AND ANSWERS

1. A channel is designed to carry a discharge of 20 m$^3$/s with Manning’s $n = 0.015$ and bed slope of 1 in 1000 (for trapezoidal channel side slope $M = 1/\sqrt{3}$). Find the channel dimensions of the most efficient section if the channel is
   (i) trapezoidal
   (ii) rectangular.  
   (AUC Nov/Dec 2010)

Given Data:

\[ Q = 20 \text{ m}^3/\text{s} \]
\[ S = \frac{1}{1000} \]
\[ m = \frac{1}{\sqrt{3}} \]
\[ n = 0.015 \]

Solution

To find
1. Dimension of a trapezoidal channel
2. Dimension of a rectangular channel

Step 1: Dimension of a trapezoidal channel:

\[ \frac{b + 2my}{2} = y \sqrt{m^2 + 1} \]
\[ b + 2 \times \frac{1}{\sqrt{3}} \times y = 2y \sqrt{m^2 + 1} \]
\[ b + 1.55 \ y = 2.309 \ y \]

\[ b = 0.759 \ y \]

\[ A = b + my \ y \]
\[ = \left( 0.759 + \frac{1}{\sqrt{3}} \ y \right) \ y \]
\[ A = 1.731 \text{ m}^2 \]
we know that \( R = \frac{y}{2} \)

\[
Q = \frac{1}{n} R^\frac{2}{3} S^\frac{1}{3} A
\]

\[
20 = \frac{1}{0.015} \left( \frac{y}{2} \right)^{\frac{2}{3}} \left( \frac{1}{1000} \right)^{\frac{1}{3}} * 1.732 \ y^2
\]

\[
= y^{\frac{8}{3}} * 1.303
\]

\[
y = \left( \frac{20}{1.303} \right)^{\frac{3}{8}}
\]

\[
y = 2.784 \ m
\]

\[
b = 0.759 \ y
\]

\[
= 0.759 * 2.784
\]

\[
= 3.213 \ m
\]

\[
b = 3.213 \ m \quad y = 2.784 \ m
\]

**Step 2: Dimension of an Rectangular channel:**

\[
b = 2y, \quad R = \frac{y}{2}
\]

\[
A = b \times y
\]

\[
A = 2y^2
\]

\[
\therefore b = 2y
\]

\[
A = b \times y
\]

\[
= 2y \times y = 2y^2
\]
\[ Q = \frac{1}{n} R^3 S^3 \]

\[ 20 = \frac{1}{0.015} \left( \frac{y}{2} \right)^\frac{2}{3} \left( \frac{1}{1000} \right)^\frac{1}{3} 2y^2 \]

\[ 20 = \frac{1}{0.015} \left( \frac{y}{2} \right)^\frac{2}{3} \left( \frac{1}{1000} \right)^\frac{1}{3} 2y^2 \]

\[ 20 = 1.506 y^\frac{8}{3} \]

\[ 13.28 = y^\frac{8}{3} \]

\[ y = 13.28^\frac{3}{8} \]

\[ = 2.638 \text{ m} \]

\[ b = 2y \]

\[ = 2 \times 2.638 \]

\[ = 5.276 \text{ m} \]

\[ b = 5.276 \text{ m} \quad y = 2.638 \text{ m} \]
2. A V-shaped open channel of included angle 90° conveys a discharge of 0.05 m³/s when the depth of flow at the center is 0.225 m. Assuming that $C = 50 \text{ m}^{1/2}/\text{s}$ in the Chezy's equation, calculate the slope of the channel. (AUC Apr/May 2010)

**Given Data:**

\[
\begin{align*}
Q &= 0.05 \text{ m}^3/\text{s} \\
C &= 50 \\
m &= 1.0 \\
y &= 0.255 \text{ m} \\
\theta &= 45^\circ
\end{align*}
\]

**Solution**

To find Slope of the given Channel

**Step 1: Slope of the Trapezoidal Channel:**

\[
Q = AC \sqrt{RS}
\]

\[
R = \frac{A}{P}
\]

\[
A = 2 \times \frac{1}{2} \times 0.225 \times 0.225
\]

\[
= 0.0506 \text{ m}^2
\]

\[
P = 2BD = 2 \times 0.318
\]

\[
= 0.636 \text{ m}
\]

\[
R = \frac{A}{P} = \frac{0.0506}{0.636} = 0.079 \text{ m}
\]
Substitute in equation 1

\[ 0.05 = 0.0506 \times 50 \times \sqrt[3]{0.079 \times S} \]

Square Root on both roots

\[ 0.0025 = (0.0506)^2 \times 2500 \times 0.079 \times S \]

\[ S = 4.923 \times 10^{-3} \]

\[ S = \frac{1}{204} \]

3. Calculate the dimensions of the rectangular cross-section of an open channel which requires minimum area to convey 10 m$^3$/s. The slope being in 1500. Take the Manning’s ‘N’ as 0.013.

(AUC Apr/May 2010)

**Given Data:**

\[ Q = 10 \text{ m}^3/\text{s} \]
\[ S = \frac{1}{1500} \]
\[ n = 0.013 \]

**Solution**

To find 1. Dimension of an Rectangular Channel:

1. \( R = \frac{y}{2} \) (For an Economical Channel)

2. \( b = 2y \)

\[ Q = AV \]

where

\[ V = \frac{1}{n} R^2 S^{\frac{1}{2}} \]
we can get
\[ Q = \frac{1}{n} R^3 S^{\frac{1}{3}} \times A \]

\[ 10 = \frac{1}{0.013} \left( \frac{y}{2} \right)^\frac{2}{3} \left( \frac{1}{1500} \right)^\frac{1}{2} \times by \]

\[ 10 = \frac{1}{0.013} \times \frac{y^2}{2^3} \times 0.0258 \times 2y^2 \]

\[ 10 = 2.502 \, y^8 \]

\[ y^3 = 3.995 \, \frac{3}{8} \]

\[ y = 1.68 \, m \]

\[ b = 2 \, y \]

\[ = 2 \times 1.68 \]

\[ = 3.36 \, m \]

dimension of the given rectangular channel
\[ b = 3.36 \, m , \, y = 1.68 \, m \]
4. A power canal of trapezoidal section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given, discharge equal to 14 m$^3$/s, bed slope 1/2500, Manning’s $n = 0.02$. (AUC Apr/May 2011)

**Given Data:**

\[
Q = 14 \text{ m}^3 / \text{s} \\
i = \frac{1}{2500} \\
n = 0.02
\]

**Solution**

To find 1. Dimension of an Trapezoidal Channel:

Her “m” value is not given, So the best side slope of METS has to be taken as “m”

\[
m = \frac{1}{\sqrt{3}}
\]

\[
\frac{b + 2nd}{2} = d \sqrt{1 + n^2}
\]

\[
\frac{b + 2 \times \frac{1}{\sqrt{3}} \times d}{2} = d \sqrt{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2d}{\sqrt{3}}
\]

Area of the trapezoidal section

\[
A = b + nd \ d
\]

\[
A = \left(\frac{2d}{\sqrt{3}} + \frac{1}{\sqrt{3}} \ d\right) \ d
\]

hydraulic an depth

\[
C = \frac{1}{N} \ m^5
\]
\[ Q = AC \sqrt{mi} \]

\[ Q = \sqrt[3]{d^2} \times \frac{1}{N} \ m^{\frac{1}{3}} \times \sqrt{m} \times \frac{1}{2500} \]

\[ = \sqrt[3]{d^2} \times \frac{1}{0.02} \ m^{\frac{1}{3}} \times \sqrt{1} \times \frac{1}{2500} \]

\[ = 1.732 \ d^2 \ m^3 \]

we can get

\[ 14 = 1.732 \ d^2 \times \left( \frac{d}{2} \right)^{\frac{2}{3}} = \frac{1.732}{2^{\frac{2}{3}}} \times d^{\frac{8}{3}} \]

\[ d^{\frac{8}{3}} = \frac{14}{1.09} = 12.84 \]

\[ d = 12.84 \times 2^{\frac{3}{8}} = 2.605 \ m \]

\[ b = \frac{2d}{\sqrt{3}} \]

\[ = \frac{2 \times 2.605}{\sqrt{3}} \]

\[ = 3.008 \ m \]

dimension of the given trapezoidal channel

\[ b = 3.008 \ m, \ d = 2.605 \ m \]
5. Determine the most economical section of rectangular channel carrying water at the rate of 0.6 cumecs. The bed slope is 1 in 2000. Assume Chezy’s constant C = 50.

(AUC Apr/May 2012)

Given Data:

\[ Q = 0.6 \text{ m}^3/\text{s} \]
\[ i = \frac{1}{2000} \]
\[ C = 50 \]

Solution

To find

1. Determine the most economical channel of the rectangular section:

\[ b = 2y \quad , \quad R = \frac{y}{2} \]
\[ A = by \]
\[ = 2y \times y = 2y^2 \]

\[ Q = AC \sqrt{mi} \]

\[ 0.6 = 2y^2 \times 50 \frac{\sqrt{\frac{y}{2}} \times \frac{1}{2000}}{\sqrt{\frac{2}{y}}} \]

\[ = 2y^2 \times 50 \times \left(\frac{y}{2}\right)^{\frac{1}{2}} \times 0.016 \]
\[ y \frac{1}{2} y^2 = 0.375 \]
\[ y^2 = 0.375 \]
\[ y = 0.675 \text{ m} \]

\[ b = 2y = 1.35 \text{ m} \]

Dimension of the most Economical Rectangular Channel \( b=1.3 \text{ m} \) and \( y = 0.675 \text{ m} \)
6. The bed width of a trapezoidal channel section is 40 m and the side slope is 2 horizontal to 1 vertical. The discharge in the canal is 60 cumecs. The Manning’s ‘n’ is 0.015 and the bed slope is 1 in 5000. Determine the normal depth

(AUC Apr/May 2012)

Given Data:

\[
\begin{align*}
Q &= 60 \text{ m}^3/\text{s} \\
S &= \frac{1}{2000} \\
m &= 2 \\
b &= 40 \text{ m} \\
n &= 0.015
\end{align*}
\]

Solution

To find 1. Determine the normal depth

\[
\begin{align*}
A &= b + m y_0 y_0 \\
&= 40 + 2 y_0 y_0 \\
P &= b + 2 y_0 \sqrt{1 + m^2} \\
&= 40 + 2 y_0 \sqrt{1 + 2^2} \\
&= 40 + 4.47 y_0 \\
R &= \frac{A}{P} = \frac{40 + 2 y_0 y_0}{40 + 4.47 y_0} \\
Q &= \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} A \\
60 &= \frac{1}{0.015} \times \left( \frac{40 + 2 y_0 y_0}{40 + 4.47 y_0} \right)^{\frac{2}{3}} \times \left( \frac{1}{5000} \right)^{\frac{1}{2}} \times 40 + 2 y_0
\end{align*}
\]
\[63.64 = \left( \frac{40 + 2y_0 - y_0}{40 + 4.47y_0} \right)^{\frac{2}{3}} \times 40y_0 + (2y_0)^2\]

Substitute in (1)

\[y_0 = 1 \text{ m} \quad ; \quad 63.64 = 40.43 \text{ m}\]

\[y_0 = 2 \text{ m} \quad ; \quad 63.64 = 130.13 \text{ m}\]

\[y_0 = 3 \text{ m} \quad ; \quad 63.64 = 79.99 \text{ m}\]

\[y_0 = 4 \text{ m} \quad ; \quad 63.64 = 71.20 \text{ m}\]

\[y_0 = 5 \text{ m} \quad ; \quad 63.64 = 62.85 \text{ m}\]

\[\therefore \text{ normal depth } y_0 = 1.30 \text{ m}\]

7. A rectangular channel of width 15m has abed slope of 0.00075 and Manning's \( n = 0.016 \). Compute the normal depth to carry a discharge of \( 50 \text{ m}^3/\text{s} \)?

**Given Data:**

\[Q = 60 \text{ m}^3/\text{s}\]

\[S_0 = 0.00075\]

\[b = 50 \text{ m}^3/\text{sec}\]

\[n = 0.016\]

**Solution**

**To find 1. Determine the normal depth:**

\[y_0 = \left( \frac{q \times n}{\sqrt{S_0}} \right)^{\frac{3}{2}}\]

where

\[q = \frac{Q}{B} = \frac{50}{15} = 3.333 \text{ m}^2/\text{sec}\]
\[ y_0 = \left( \frac{3.33 \times 0.016}{\sqrt{0.00075}} \right)^{\frac{3}{5}} \]

\[ y_0 = 1.47 \text{ m} \]

verification

\[ \frac{y_0}{B} = \frac{1.47}{15} = 0.09 > 0.02 \]

So we can check it in another way

\[ \frac{Q_o}{\sqrt{s_0}} = AR^2 \]

\[ \left( \frac{y_0}{B} \right)^{\frac{5}{3}} \times B^{\frac{8}{3}} = By_0 \times \left( \frac{By_0}{B + 2y_0} \right)^{\frac{2}{3}} \]

\[ \left( \frac{1.47}{15} \right)^{\frac{5}{3}} \times (15)^{\frac{8}{3}} = 15 \times 1.47 \times \left( \frac{15 \times 1.47}{15 + 2 \times 1.47} \right)^{\frac{2}{3}} \]

\[ 25.3305 = 25.3005 \]

Hence Proved
Problem : 8.0  Find the velocity of flow and rate of flow of water through a rectangular channel of 5m wide and 2.5m deep when it is running full. The channel has a bed slope of 1 in 1250. Take chezy's constant as 50.

Solution

Given

\[ b = 5 \text{ m} \]
\[ d = 2.5 \text{ m}. \]

\[ \therefore \ A = 5 \times 2.5 = 12.5 \text{ m}^2 \]
\[ P = 5 + 2 \times 2.5 = 10 \text{ m}. \]

\[ \therefore \ m = \frac{A}{P} = \frac{12.5}{10} = 1.25 \text{ m}. \]

Velocity of flow \[ V = C \sqrt{\frac{m}{1}} = 50 \sqrt{1.25} \times \frac{1}{1250} \]

\[ = 1.58 \text{ m/s}. \]

Discharge \[ Q = AV = 12.5 \times 1.58 = 19.75 \text{ m}^3/\text{s} \]

Problem : 9  Find the slope of the bed of a rectangular channel of width 6m and depth of flow 3m and rate of flow 27.11 m$^3$/s. Take chezy's constant \( C = 55 \).

Solution

Given

\[ b = 6 \text{ m} \]
\[ d = 3 \text{ m} \]
\[ Q = 27.11 \text{ m}^3/\text{s} \]
\[ C = 55 \]

\[ A = b \times d = 18 \text{ m}^2 \]
\[ P = b + 2d = 6 + 2 \times 3 = 12 \text{ m}. \]

\[ \therefore \ m = \frac{A}{P} = \frac{18}{12} = 1.5. \]

Slope = i.

using equation 2.2

\[ Q = A C \sqrt{\frac{m}{i}} \]

\[ \therefore 27.11 = 18 \times 55 \sqrt{1.5 \times i} \]
Squaring both sides

\[ 27.11^2 = 18^2 \times 55^2 \times 1.5 \times i \]

\[ \therefore i = \frac{27.11^2}{18^2 \times 55^2} \times \frac{1}{1.5} = \frac{734.9521}{324 \times 3025 \times 1.5} \]

= 2000.8003.

or say 1 in 2000.

\[ \therefore \text{Bed slope is 1 in 2000.} \]

Problem :10 100 lps is flowing in a rectangular flume of width 600 mm and of flow 300 mm. With chezy's constant C as 56. Find the bottom slope for the flume to have uniform flow. Also find the conveyance K of the flume.

Solution:

Given: \( Q = 100 \text{ lps.} = \frac{100}{1000} = 0.1 \text{ m}^3/\text{s.} \)

\[ b = 600 \text{ mm} = 0.6 \text{ m} \]

\[ d = 300 \text{ mm} = 0.3 \text{ m} \]

\[ A = b \times d = 0.6 \times 0.3 = 0.18 \text{ m}^2 \]

\[ P = b + 2d = 0.6 + 2 \times 0.3 = 1.2 \text{ m.} \]

\[ m = \frac{A}{P} = \frac{0.18}{1.2} = 0.15 \text{ m.} \]

Using equation (2.2): \( Q = AC \sqrt{mi} \)

\[ 0.10 = 0.18 \times 56 \sqrt{0.15 \times i} \]

Solving \( i = \frac{1}{1524} \)

Slope of the bed is 1 in 1524

C) Conveyance K of the channel

Conveyance K of a channel is \( AC \sqrt{m} = K. \)

\[ K = 0.18 \times 56 \times \sqrt{0.15} = 3.9 \text{ m}^3/\text{s} \]
Problem: 10  A Trapezoidal channel has side slopes 1:1 and has a bed width of 5m. If the flow depth is 2m, find the velocity of flow and discharge through the channel. Take bed slope as 1 in 2000 and chezy's constant c = 50.

Solution

\[ b = 5 \text{m} \]
\[ d = 2 \text{m} \]

Side slope \[ = d \sqrt{1 + 1^2} \]
\[ = 2 \sqrt{2} \]
\[ P = 5 + 2 \times 2 \sqrt{2} \]
\[ = 5 + 4 \sqrt{2} = 5 + 4 \times 1.414 = 5 + 5.656 \]
\[ = 10.656 \]
\[ A = 2 \left( \frac{9 + 5}{2} \right) = 14. \]

\[ m = \frac{A}{P} = \frac{14}{10.656} = 1.3138 \]

\[ V = 50 \sqrt{\frac{14}{10.656} \times \frac{1}{2000}} = 1.2815 \text{ m/s}. \]

\[ Q = AV = 14 \times 50 \sqrt{\frac{14}{10.656 \times 2000}} = 17.94 \text{ m}^3/\text{s} \]

Problem: 11  Find the velocity and discharge of water through a Trapezoidal channel whose base width = 6 m, depth of flow = 4 m, and side slopes 1:1½. The bed slope is 1 in 2000 and chezy’s constant c = 55.

Solution:

Given \[ b = 6 \text{m} \]
\[ d = 4 \text{m} \]

Length of sides \[ = \sqrt{b^2 + d^2} \]
\[ = \sqrt{36 + 16} = \sqrt{52} = 7.21 \text{ m.} \]
\[ P = 6 + 2 \sqrt{52} = 20.42 \]
\[ A = \left( \frac{6 + 18}{2} \right) \times 4 = 48 \text{ m}^2 \]
\[ m = \frac{A}{P} = \frac{48}{20.42} = 2.35 \text{ m} \]

\[ V = c \sqrt{mi} \quad ; \quad Q = AV = AC \sqrt{mi} \]

\[ \therefore Q = 48 \times 55 \sqrt{\frac{48}{\sqrt{20.42}} \times \frac{1}{2000}} = 48 \times 55 \sqrt{2.35 \times \frac{1}{2000}} \]

\[ = 90.5 \text{ m}^3/\text{s} \]

**Problem 12** Determine the rate of flow through a 4m wide rectangular channel and depth of flow 2m and bed slope 1 in 10000; take \( n = 0.01 \)

**Solution:**

Area of flow \( A = b \times d \)

\[ = 4 \times 2 = 8 \text{ m}^2 \]

Wetted perimeter \( = b + 2d = 4 + 2 \times 2 = 8 \text{m}. \)

\[ \therefore R = \frac{A}{P} = \frac{8}{8} = 1 \text{ m} \]

\[ \therefore Q = \frac{1}{n} AR^{2/3} S^{1/2} \]

\[ = \frac{1}{0.01} \times 8 \times 1^{2/3} \left( \frac{1}{10000} \right)^{1/2} \]

\[ = \frac{1}{0.01} \times 8 \times 1 \times \frac{1}{100} \]

\[ = 8 \text{ m}^3/\text{s}. \]

**Problem 13** A flume whose \( n = 0.01 \) is in the form of an inverted equilateral triangle carrying water to a depth of 1.0m. The slope of the bed \( = 0.01 \). Find the discharge.

**Solution:**

\[ \cos 30^\circ = \frac{\sqrt{3}}{2} \]

Side length \( L; \) depth of flow \( d \)

Then \( \frac{d}{L} = \cos 30^\circ = \frac{\sqrt{3}}{2} \)

\[ \therefore L \sqrt{3} = 2d \]

\[ \therefore L = \frac{2d}{\sqrt{3}} = \frac{2 \times 1}{\sqrt{3}} = \frac{2.0}{1.732} = 1.155 \text{ m} \]
\[ P = 2L = \frac{4}{1.732} = 2.3095 \]

\[ A = \frac{1}{2} L \times d = \frac{1}{2} \cdot \frac{2.0}{1.732} \times 1 = \frac{1}{1.732} = 0.5774 \text{ m}^2 \]

\[ R = \frac{A}{P} = \frac{1}{1.732} \times \frac{1.732}{4} = 0.25 \text{ m} \]

Mean velocity \[ V = \frac{1}{n} \times \frac{R^{2/3}}{S_0^{1/2}} \]

\[ = \frac{1}{0.01} \times \frac{0.25^{2/3}}{0.01^{1/2}} \\
= \frac{0.25^{2/3}}{0.01^{1/2}} = 3.97 \text{ m/s} \]

\[ Q = A \times V = \frac{1}{\sqrt{3}} \times \frac{0.25^{2/3}}{0.01^{1/2}} = 2.29 \text{ m}^3/\text{s} \]

**Problem 14** A trapezoidal channel of base width 4m and depth 2.0m has side slopes at an angle 45° to the horizontal. If the bed slope \( S_0 = 0.001 \). When the flow is 12 m find Manning’s ‘n’ and boundary shear stress.

![Fig. 2.11](image)

Area of flow \[ = \left( \frac{4 + 8}{2} \right) \times 2 = 12 \text{ m}^2 \]

Wetted perimeter \[ = P = 4 + 2 \times \sqrt{8} \]

\[ = 4 + 4 \sqrt{2} = 4 + 4 \times 1.414 \]

\[ = 9.656 \text{ m} \]
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\[ R = \frac{A}{P} = \frac{12}{9.656} = 1.2428 \text{ m} \]
\[ V = \frac{Q}{A} = \frac{12}{12} = 1 \text{ m/s} \]
\[ V = \frac{1}{n} R^{2/3} S^{1/2} \]
\[ V = \frac{1}{n} R^{2/3} S^{1/2} \]
\[ n = \frac{R^{2/3} S^{1/2}}{V} = \frac{1.2428^{2/3} \times (0.001)^{1/2}}{1} \]
\[ \tau_0 = \rho g R S = 1000 \times 9.81 \times 1.2428 \times 0.001 \]
\[ = 9.81 \times 1.2428 \]
\[ = 12.19 \text{ N/m}^2 \]

**Problem:** 15  A wide rectangular channel 30 m wide has uniform flow and discharge 10 m³/s. Bed slope is 0.0009. Determine the normal depth of flow if \( n = 0.015 \)

**Solution**

Given \( Q = 10 \text{ m}^3/\text{s}; \ n = 0.015 \)
\( B = 30 \text{ m.} \)
\( S_0 = .0009 \)
\( q = \frac{10 \text{ m}^3/\text{s}}{30 \text{ m}} = 0.333 \text{ m}^3/\text{s}/\text{m.} \)
\[ y_0 = \left[ \frac{qn}{\sqrt{S_0}} \right]^{3/5} = \left[ \frac{0.333 \times 0.015}{\sqrt{0.0009}} \right]^{3/5} = \left( \frac{0.333 \times 0.015}{0.03} \right)^{3/5} \]
\[ = (0.167)^{3/5} \text{ m} \]

\[ : \text{ The normal depth is 0.34 m} \]

Verify \( \frac{y_0}{B} = \frac{0.34}{30} = < 0.02 \text{ OK.} \)

If \( \frac{y_0}{B} > 0.02, \quad \frac{Q_i}{\sqrt{S_0}} = AR^{2/3} \)
Problem: 16  A sewer pipe 2.0 m dia is laid on a slope of 0.0009 find the normal depth of flow where discharge is 2 m³/s assuming n = 0.015

Solution:
\[
\frac{AR^{2/3}}{D^{5/3}} = \frac{Q_n}{\sqrt{S_0}} \frac{D^{8/3}}{D^{8/3}} = \frac{2.0 \times 0.015}{\sqrt{0.0009 \times (2.0)^{8/3}}}
\]
\[
\frac{Q_n}{\sqrt{S_0}} = \frac{A^{5/3}}{B^{5/3}} = \frac{D^{10/3}}{8^{5/3}} \left( \frac{2 \theta - \sin 2 \theta}{D^{2/3}} \right), \cos \theta = \frac{r_0 - y_0}{r_0} = 1 - \frac{2 y_0}{D}
\]
\[
\therefore y_0
\]

Problem: 17  A rectangular channel of width 6m is having a bed slope of 1 in 2000. Find the maximum discharge, taking value of C as 50.

Solution:
\[
y_{es} = \frac{B_{es}}{2} = \frac{6}{2} = 3 m.
\]

\[
B = 6 m
\]

\[
i = \frac{1}{2000}
\]

\[
C = 50
\]

\[
m = \frac{A}{P} = \frac{B y_{es}}{B + 2 y_{es}} = \frac{6 \times 3}{6 + 2 \times 3}
\]

\[
= \frac{18}{12} = 1.5 m.
\]

\[
\text{Discharge } Q = AC \sqrt{i} = 18 \times 50 \sqrt{1.5 \times \frac{1}{2000}}
\]

\[
= 24.65 \text{ m}^3/\text{s}
\]
Problem: 18  A rectangular channel with a bed slope of $\frac{1}{2000}$ carries water at the rate of 4.0 m$^3$/s. Find the most economical section if Manning’s $n = 0.015$.

\[ Q = \frac{1}{n} AR^{2/3} S_0^{1/2} \]

\[ Q = 4 \text{ m}^3/\text{s} \]
\[ n = 0.015 \]
\[ Q = B_{es} \times y_{es} = 2 y_{es} \times y_{es} \]
\[ = 2 y_{es}^2 \]
\[ P = B_{es} + 2 y_{es} = 2 y_{es} + 2 y_{es} = 4 y_{es} \]
\[ \therefore R = \frac{A}{P} = \frac{2 y_{es}^2}{4 y_{es}} = \frac{y_{es}}{2} \]

\[ Q = \frac{1}{n} AR^{2/3} S_0^{1/2} = \frac{1}{0.015} \times \left( \frac{y_{es}}{2} \right)^{2/3} \times \left( \frac{1}{2000} \right)^{1/2} \]
\[ 4 = \frac{1}{0.015} \times 0.5^{2/3} \times \left( \frac{1}{2000} \right)^{1/2} \times y_{es}^{2/3} \]
\[ \therefore y_{es} = \]
\[ B_{es} = 2 y_{es} = \]

Problem: 19  A power canal of trapezoidal section has to be excavated through hard soil at the least cost. Determine the dimensions of the channel given, discharge equal 15 m$^3$/s with bed slopes 1:2000 use Manning coefficients $= 0.022$

\[ Q = 15 \text{ m}^3/\text{s} \]
\[ S_0 = \frac{1}{2000} \]
\[ n = 0.022. \]

For economical section
\[ \frac{y}{x} = \tan 60 = \sqrt{3} \]
\[ x = \frac{y}{\sqrt{3}} \]

Fig. 2.22
\[ b = \text{side length} = \sqrt{y^2 + \left(\frac{y}{\sqrt{3}}\right)^2} = \sqrt{y^2 + \frac{y^2}{3}} \]

\[ \therefore \quad b = \frac{2y}{\sqrt{3}} = \text{side slope length} \]

\[ A = \frac{(2b + 2y/\sqrt{3})}{2} \times y \]

\[ = \left(\frac{4y}{\sqrt{3}} + \frac{2y}{\sqrt{3}}\right) \times y = \frac{3y^2}{\sqrt{3}} \]

\[ A = (b + x) \times y \]

\[ = \left(\frac{2y}{\sqrt{3}} + \frac{y}{\sqrt{3}}\right) \times y = \frac{3y^2}{\sqrt{3}} \]

for economical section all three sides are same.

\[ P = 3b = \frac{6y}{\sqrt{3}} = 2 \sqrt{3} \ y. \]

\[ R = \frac{A}{P} = \frac{3y^2}{\sqrt{3}} \times \frac{1}{2 \sqrt{3} \ y} = \frac{y}{2} \]

\[ Q = \frac{1}{n} \ A \ R^{2/3} \ S_0^{1/2} \]

\[ 15 = \frac{1}{0.022} \times \frac{3y^2}{\sqrt{3}} \times \left(\frac{y}{2}\right)^{2/3} \times \left(\frac{1}{2000}\right)^{1/2} \]

\[ 15 = \frac{1}{0.022} \times 1.732 \times (0.5)^{2/3} \times \left(\frac{1}{2000}\right)^{1/2} \times y^{8/3} \]

\[ \therefore \ y = 2.66 \ m \]

with \[ b = \frac{2y}{\sqrt{3}} = \frac{2 \times 2.66}{\sqrt{3}} = 3.07 \ m. \]
Problem: 20  Through a circular channel of diameter 60 cm, 150 lps is discharged at the maximum velocity. Taking C as 60 find the slope at which the pipe is to be laid.

Given:  
\[ Q = 150 \text{ lps} = 0.15 \text{ m}^3/\text{S} \]
\[ D = 60 \text{ cm} = 0.6 \text{ m} \]
\[ C = 60 \]

Slope at which pipe is to be laid = \( i \),

For maximum velocity the depth of flow is 0.81 \( D \).

\[ y_{es} = 0.81 \times 0.6 = 0.486 \text{ m} \]
\[ \theta = 128^\circ 45' = 2.247 \text{ radian.} \]
\[ m = 0.3 \times D = 0.3 \times 0.6 = 0.18 \text{ m} \]
\[ P = 2 \times R \times D = 0.6 \times 2.247 = 1.3482 \text{ m} \]

\[ m = \frac{A}{P} = 0.18 \]

\[ A = mP = 0.18 \times 1.3482 = 0.2426 \text{ m}^2 \]
\[ Q = AC \sqrt{m} \]
\[ 0.15 = 0.226 \times 60 \sqrt{0.18} \]

\[ i = \left( \frac{0.15}{5.753} \right)^2 = 0.00068 = \frac{1}{1471} \text{ Ans.} \]

1 in 1471.

Problem: 21  Find the maximum discharge of water passing through a pipe of diameter 1.5 m, laid at a slope of 1 in 1200 taking chezy's constant \( C = 55 \).

Given:

\[ i = \frac{1}{2000} \]
\[ D = 1.5 \text{ m}, \ R = 0.75 \text{ m} \]
\[ C = 55 \]

For maximum discharge \( \theta = 154^\circ \times \pi = 2.6878 \text{ radians} \)
\[ = 2 \times \frac{1.5}{2} \times 2.6875 = 4.0317 \text{ m} \]

Area of C.S. of flow \( A = R^2 \left( \theta - \frac{\sin \theta}{2} \right) \)

\[ = 0.75^2 \left[ 2.6878 - \frac{\sin 308}{2} \right] \]

\[ = 0.75^2 \left[ 2.6878 - \frac{\sin (360 - 52)}{2} \right] \]

\[ = 0.75^2 \left[ 2.6876 + \frac{\sin 52}{2} \right] \]

\[ = 1.7335 \text{ m}^2 \]

Hydraulic mean depth \( m = \frac{A}{P} = \frac{1.7335}{4.0317} = 0.4299 \text{ m} \)

\[ \therefore \text{ Maximum discharge } Q = AC \sqrt{mi} \]

\[ = 1.7335 \times 55 \sqrt{0.4299 \times \frac{1}{1200}} \]

\[ = 1.8045 \text{ m}^3/\text{s} \]

Problem: 22

A concrete circular channel of diameter 3 meter has a bed slope \( \frac{1}{1200} \) and \( C = 60 \). Calculate the velocity and discharge for the following condition (1) When velocity is maximum and (2) When discharge is maximum.

Solution:

Given: \( D = 3 \text{ m}; i = \frac{1}{1200}; C = 60 \).

(1) To find maximum velocity and corresponding discharge when velocity is maximum for max. velocity,

\[ \theta = 128^\circ 45' = \frac{128.75 \times \pi}{180} = 2.247 \text{ radian} \]

\[ P = 2R \theta = 3 \times 2.247 = 6.741 \text{ m} \]
\[ A = R^2 \left( 0 - \frac{\sin 2\theta}{2} \right) = 1.5^2 \left[ 2.247 - \frac{\sin 2 \times 128.75}{2} \right] \]
\[ = 2.25 \left[ 2.247 - \frac{\sin (257.5^\circ)}{2} \right] \]
\[ = 2.25 \left[ 2.247 - \frac{\sin (180 + 97.5^\circ)}{2} \right] \]
\[ = 2.25 \left[ 2.247 + \frac{\sin 97.5^\circ}{2} \right] \]
\[ = 2.25 \times (2.247 + 0.489) \]
\[ = 6.156 \text{ m}^2 \]

Also \[ m = \frac{A}{R} = \frac{6.156}{6.741} = 0.913 \text{ m} \]

Also \[ m = 0.3 \text{ D} \]
\[ \therefore m = 0.3 \times 3 = 0.9 \text{ m} \]

Maximum velocity \[ V = C \sqrt{m} = 60 \times \sqrt{0.9 \times \frac{1}{1200}} \text{ m/s} \cdot \]
\[ = 1.971 \text{ m/s} \]

\[ Q = AV = 6.1537 \times 1.971 \]
\[ = 12.33 \text{ m}^3/\text{s} \]

2) The maximum discharge, velocity & discharge for maximum discharge.

\[ \theta = 154^\circ = \frac{154 \times \pi}{18} = 2.6878 \text{ radian.} \]

\[ A = R^2 \left[ 0 - \frac{\sin 2\theta}{2} \right] \]
\[ = 1.5^2 \left[ 2.6878 - \frac{\sin (2 \times 154^\circ)}{2} \right] \]
\[ = 2.25 \left[ 2.6878 - \frac{\sin (360 - 308)}{2} \right] \]
\[ = 2.25 \left[ 2.6878 + \frac{\sin 52^\circ}{2} \right] \]
\[ = 2.25 \times (2.6878 + 0.394) \]
\[ = 6.934 \text{ m}^2 \]
\[ P = 2R \theta = 2 \times 1.5 \times 2.6878 \]
\[ = 8.0634 \text{ m} \]
\[ m = \frac{A}{P} = \frac{6.934}{8.0634} = 0.8599 \]

\[ \therefore V \text{ for maximum discharge} = C \sqrt{mi} \]
\[ = 60 \sqrt{0.8599 \times \frac{1}{1200}} \]
\[ = 1.6061 \text{ m/s} \]

Max. Discharge \( Q = Q_{\text{max}} = AV = 6.934 \times 1.6061 \text{ m}^3/\text{s} \)
\[ = 11.137 \text{ m}^3/\text{s} \]

Problem : 24  
A trapezoidal canal has side slopes 1 to 1. It is required to discharge 13.75 \( \text{m}^3/\text{s} \) of water with a bed gradient of 1 in 1000. If unlined, value of chezy’s \( C \) is 44. If lined with concrete, its value is 60. The cost per \( \text{m}^3 \) of excavation is four times the cost per \( \text{m}^2 \) of lining. For the canal to be the most efficient one, find whether the lined canal or the unlined canal will be cheaper. What will be dimensions of that economical canal?

Given:

\[ \begin{align*}
\text{side Slope} & = 1 \text{ to } 1 \\
\text{i.e., } m & = 1 \\
Q & = 13.75 \text{ m}^3/\text{s.}
\end{align*} \]
Slope of bed \( i = \frac{1}{1000} \)

for unlined canal \( C = 44 \)

for lined canal \( C = 60 \)

Cost per \( m^2 \) of lining = \( x \)

Cost per \( m^3 \) of excavation = \( 4x \)

Canal should be most efficient.

For most efficient channel \( \frac{A}{P} = m = \frac{y}{2} \).

Area of canal \( A = (b + y) y \)

wetted perimeter \( P = b + 2 \sqrt{2} \cdot y \)

\[
m = \frac{A}{P} = \frac{(b + y) y}{b + 2 \sqrt{2} \cdot y} = \frac{y}{2}
\]

\[
\therefore 2y (b + y) = by + 2 \sqrt{2} \cdot y^2
\]

\[
2 by + 2 y^2 = by + 2 \sqrt{2} \cdot y^2
\]

\[
by + 2 y^2 = 2 \sqrt{2} \cdot y^2
\]

\[
by = (2 \sqrt{2} - 2) y^2
\]

\[
by = 0.828 y^2
\]

\[
b = 0.828 y
\]

\[
\therefore A = (b + my) y = (b + y) y = 1.828 y^2
\]

For Unlined Canal:

\[Q = A \cdot V = A \cdot C \cdot \sqrt{m} \]

\[13.75 = 1.828 \times y^2 \times 44 \sqrt{\frac{y}{2} \cdot \frac{1}{1000}} \quad \Rightarrow \quad y = \frac{d}{2}
\]

\[
\therefore y^{3/2} = 7.6451; \quad \therefore y = 7.6452^{2/5} = 2.256 \text{ m.}
\]

\[
b = 0.828 y = 0.828 \times 2.256 = 1.868 \text{ m}
\]
Now cost of excavation for one meter length of canal

\[ \text{Cost} = (\text{Volume of canal}) \times \text{cost per } \text{m}^3 \text{ of excavation} \]

\[ = (\text{Area of C.S } \times 1 \text{ m length}) \times 4x. \]

\[ = 1.828 \times 2.256^2 \times 1 \times 4x \]

\[ = 1.828 \times 2.256^2 \times 1 \times 4x \]

\[ \therefore \text{Cost of unlined canal} = 37.215x \]

For lined canal:

Value of \( C = 60 \)

\[ Q = AC \sqrt{mi} \]

\[ 13.75 = 1.828 \times y^2 \times 60 \sqrt{\frac{y}{2} \times \frac{1}{1000}} \left( m = \frac{d}{2} \right) \]

\[ y^{5/2} = \frac{13.75 \times \sqrt{2000}}{1.828 \times 60} = 5.606 \]

\[ \therefore y = 5.606^{2/5} = 1.992 \text{ m} \]

\[ \therefore b = 0.828 \times y = 0.828 \times 1.992 = 1.649 \text{ m} \]

Here cost of excavation + cost of living is to be considered.

Cost of excavation / m = Volume of canal/m \times \text{cost per m}^3 \text{ of excavation}

\[ = \text{Area of C.S } \times 1 \times 4x \]

\[ = (b + my) \times 1 \times 4x \]

\[ = (1.649 + 1.992) \times 1.992 \times 4x \]

\[ = 29.01x \]
25. **Explain the concept of uniform flow with all the details**

**Definition and concept:**

If the flow velocity at a given instant of time does not vary within a given length of channel, then the flow is called *uniform flow*. Uniform flow is a unique flow condition that is not extremely common in natural streams and channels. In order for uniform flow to exist, the depth, cross-sectional area, velocity, and flow at each section of a channel reach must be constant. In addition, the energy line, water surface, and channel bed must be parallel. Because uniform unsteady flow does not exist, it is always classified as steady uniform flow. For practical purposes uniform flow is often assumed in order to compute the discharge of a natural stream.

**Concept of Uniform Flow**

1. The depth average flow velocity (integrated over depth), area of flow cross-sections are everywhere constant along the channel.

2. The energy grade line $S_f$, water surface slope $S_w$ and channel bed slope $S_0$ are all parallel, i.e. $S_f = S_w = S_0$

When the flow enters into a channel, the boundary layer grows up to free surface. The region for a mild channel can be divided into three zones viz., initial transitory zone in the entrance. Flow changes from the uniform flow to critical flow in the transitory zone at exit in mild channel. The boundary layer as it grows along the channel at the entrance emerges to the free surface at a certain distance from the entry point. This zone is called entry transition zone.
If the bed slope is critical slope, then the transitory zone in the entrance only exists. The uniform flow extends till the flow terminates and exits as a jet at critical depth. This flow is known as critical uniform flow. The free surface will be undulating with waves moving at \( C = \sqrt{gy} \). In the case of steep channel, the flow enters either through a hydraulic drop or at uniform flow depth. This has an initial transitory zone with an S2 type of varied flow curve. The flow emerges from the steep slope at uniform flow depth \((y_n>y_c)\).

![Diagram showing boundary layer growth in open channel with an ideal entry condition.](image)

**Figure shows Boundary layer growth in open channel with an ideal entry condition.**

1. Uniform flow has constant velocity at every point on channel section. The flow depth in uniform flow is known as normal depth.

2. Uniform flow is always steady, as unsteady uniform flow practically never occurs. However, turbulent uniform flow does occur.
3. When flow occurs in an open channel, resistance is encountered by water as it flows downstream. This resistance is generally counteracted by the components of gravity forces acting on the body of water in direction of motion. A uniform will be developed if resistance is balanced by the gravity forces.

4. If the water enters the channel slowly, the velocity and hence the resistance are small and the resistance is outbalanced by gravity forces, resulting in an accelerating flow in the upstream reach. The resistance will gradually increase until a balance between resistance and gravity forces is reached. At this moment and afterwards the flow becomes uniform.

5. The upstream reach that is required for the establishment of uniform flow is known as transitory zone. In this zone, the flow is accelerating and varied.

6. If the channel is shorter than the transitory length required by given conditions, uniform flow cannot be attained.

7. The length of transitory zone depends on discharge and on the physical conditions of the channel such as entrance condition, shape, slope and roughness.

8. Depth of flow required for uniform flow is known as normal depth.

9. A prismatic channel carrying a certain discharge with constant velocity is an example of uniform flow.

26. How do you determine velocity of flow in open channel?  

   (AUC Apr/may 2012)

   For hydraulic computations, the mean velocity of a uniform flow in open channels is usually expressed approximately by a so-called uniform flow formula.

   For practical purposes, the flow in a natural channel may be assumed uniform under normal conditions, that is, if there are no flood flows or markedly varied flow caused by channel irregularities.

   In uniform flow since the velocity of flow does not change along the length of the channel, acceleration is zero. Hence, the sum of the components of all the external forces in the direction of flow must be equal to zero.
27. Explain the computation of uniform flow using Manning’s and Chezy’s method.

(AUC Nov/Dec 2010)

**Case (i) Chezy’s Equation**

Consider a short reach of channel of length L in which a uniform flow has been established. Let us say, Fm is the force causing the motion which is the gravity force of water body between the sections of length, L acting along the flow direction. Let A be the cross sectional area of the flow which is constant throughout the length L of the channel.

Here, θ is the bed slope; is the unit weight of the fluid. For uniform flow Therefore, above equation can be written as where, Rh= hydraulic radius; S= bottom slope of the channel. The above equation gives the bed shear (strictly speaking, it is the average shear stress on the wetted area) under uniform flow condition.

![Definition sketch of flow in open channel](image)

**Case (ii) Manning’s Equation**

In 1889 the Irish Engineer Robert Manning presented a formula, which was later modified to its present well known form

\[ V = \frac{1}{n} R^{2/3} S^{1/2} \]

Where V is the mean velocity, R is hydraulic radius and S is slope of energy line and n is coefficient of roughness. Manning formula has become the most widely used of all uniform flow formula. Comparing this with Chezy equation we get

\[ C = \frac{1}{n} R^{1/6} = \sqrt{\frac{8g}{f}} \]

\[ f = \left( \frac{n^2}{R^{1/3}} \right) (8g) \]
The Manning's formula is most convenient and commonly used for practical purpose. However, it has a limitation that it does not adequately represent the resistance in situations where the Reynolds number effect is predominant.

<table>
<thead>
<tr>
<th>Lined Canals</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement plaster</td>
<td>0.011</td>
</tr>
<tr>
<td>Untreated gunite</td>
<td>0.016</td>
</tr>
<tr>
<td>Wood, planed</td>
<td>0.012</td>
</tr>
<tr>
<td>Wood, unplaned</td>
<td>0.013</td>
</tr>
<tr>
<td>Concrete, trowled</td>
<td>0.012</td>
</tr>
<tr>
<td>Concrete, wood forms, unfinished</td>
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</tr>
<tr>
<td>Rubble in cement</td>
<td>0.020</td>
</tr>
<tr>
<td>Asphalt, smooth</td>
<td>0.013</td>
</tr>
<tr>
<td>Asphalt, rough</td>
<td>0.016</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Natural Channels</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel beds, straight</td>
<td>0.025</td>
</tr>
<tr>
<td>Gravel beds plus large boulders</td>
<td>0.040</td>
</tr>
<tr>
<td>Earth, straight, with some grass</td>
<td>0.026</td>
</tr>
<tr>
<td>Earth, winding, no vegetation</td>
<td>0.030</td>
</tr>
<tr>
<td>Earth, winding with vegetation</td>
<td>0.050</td>
</tr>
</tbody>
</table>

28. What is meant by roughness co-efficient. How do you determine the Roughness Coefficients..Explain in detail

In applying the Manning formula, the greatest difficulty lies in the determination of the roughness coefficient n; for there is no exact method of selecting the value of n.

For proper determination of roughness coefficient, four approaches can be used.
i. To understand the factors that affect the value of n and thus to acquire a basic knowledge of problem and narrow the wide range of guesswork.

ii. To consult a table of typical n values for channels of various forms.

iii. To examine and become acquainted with the appearance of some typical channels whose roughness coefficient are known.

iv. To determine value of n by an analytical procedure based on theoretical velocity distribution in channel cross section and on the data of either velocity or roughness measurement.

There are certain factors which affect Manning’s roughness coefficient:

**Surface roughness:**

The roughness on the channel boundary provides a critical point of reference in estimating n.

If the material of the channel surface is fine, the value of n is low and is relatively unaffected by changes in depth of flow.

When the material is coarse like that of gravel and/or boulders the value of n is larger and may vary significantly with the depth of flow.

**Vegetation:**

Vegetation in the channel retards the flow and thus increases n: in general, the relative importance of vegetation on n is a strong function of the depth of flow and the height, density, distribution and type of vegetation.

**Channel irregularities:**

Channel irregularities refer to the variations in channel cross section, shape and wetted perimeter along the length of the channel.

In natural channels, such irregularities are usually the result of deposition or scour. In general, gradual variations have rather insignificant effect on n, while abrupt changes can result in a much higher n value that would be expected from the consideration of only the surface roughness in the channel.

**Channel alignment:**

While curves of large radius without frequent changes in the direction of curvature offer relatively little resistance to flow and affects n where, as severe meandering with curves of small radius will significantly increase n values.
Sedimentation and scouring:
In general, active sedimentation and scouring yields channel variation which results in an increased value of n.

Type of bed form has a significant change in n value.

Stage and discharge:

The n value for most channels tends to decrease with an increase in the stage and discharge.

This is the result of irregularities which have a crucial impact on the value of n at low stages when they are uncovered.

However, n value may increase with increasing stage and discharge if the banks of the channel are rough, grassy, or brush-covered, or if the stage increases sufficiently to cover the flood plain.

On flood plains, the value of n usually varies with the depth of submergence. In such cases it is necessary to compute a composite value of n.

Few aids are available that help in selecting an appropriate values of n for a given channel. They are

- Photographs of selected typical reaches of canals and their description and measured values of n. By comparing the channel under consideration with the appropriate figure one can select the value of Manning's n.

- A comprehensive list of various types of channels and their description with the range of values of n.

There are many empirical formulae presented to estimate Manning's n. In these empirical formulae n is related to bed particle size.
29. Explain the method of Determination Of Normal Depth And Velocity
(AUC Apr/may 2012)

**Determination Of Normal Depth**

Manning’s formula \( Q = \frac{1}{n} AR^{2/3}S_0^{1/2} \) can also be written as \( Q = KS_0^{1/2} \) where \( K = \frac{1}{n} AR^{2/3} \) is known as channel conveyance. The expression \( A + R^{2/3} \) is called as section factor (Z) for uniform flow computation.

Conveyance can be defined as discharge capacity of the channel per unit longitudinal slope. It can be seen that \( K = f(y, B, \frac{1}{n}) \)

In case of channel with either constant or increasing top width, the conveyance \( K \) is a single valued function of \( y \), provided that side slope (\( m \) horizontal to 1 vertical) \( m \geq 0 \). Such channels are known as channel of first kind. For example, trapezoidal, rectangle, triangular. As \( K \) is single valued function of \( y \) in channels of first kind, they have a unique depth for a given discharge known as normal depth. Normal depth is depth of flow for uniform flow. The channels with closing top width can be classified as channels of second kind. For example, circular section. For such channels \( K \) may not be a single valued function of depth. Therefore, in such channels for a particular discharge there can be two depths possible.

**Determination Of Velocity**

For hydraulic computations, the mean velocity of a uniform flow in open channels is usually expressed approximately by a so-called uniform flow formula.

For practical purposes, the flow in a natural channel may be assumed uniform under
normal conditions, that is, if there are no flood flows or markedly varied flow caused by channel irregularities.

In uniform flow since the velocity of flow does not change along the length of the channel, acceleration is zero. Hence, the sum of the components of all the external forces in the direction of flow must be equal to zero.

30. Explain the Most Economical rectangular and Trapezoidal channel Section

It is known that the conveyance of a channel section increases with increases in hydraulic radius or with decrease in the wetted perimeter. From hydraulics viewpoint, therefore, the channel section having the least wetted perimeter for a given area has the maximum conveyance; such a section is known as the best hydraulic section. Of all the possible open channel sections, the semicircular shape has the least amount of perimeter for a given area. Relationship between various geometric elements to form an efficient section can be obtained as follows:

RECTANGULAR CHANNEL
31. Explain in detail about non-erodible channel's Non Erodible channels

Most lined channels and built up channels can withstand erosion satisfactorily and are therefore, considered non-erodible. Unlined channels are generally erodible, except those excavated in firm foundations. In designing non erodible channels, such factors as the maximum permissible velocity and the permissible tractive force are not criteria to be considered. The designer simply computes the dimensions of the channel by a uniform flow formula and then decides the final dimensions on the basis of hydraulic efficiency, or empirical rule of best section, practicability and economy.

The factors to be considered in design are:
i. The kind of material forming the channel body, which determines the roughness coefficient.
ii. The minimum permissible velocity, to avoid deposition if the water carries silt
iii. Channel bottom slope and side slopes
iv. Freeboard
v. The most efficient section

The non erodible materials used to form the lining of channel and body of a built-up channel include concrete, stone masonry, steel, cast iron, timber, glass. The selection of the material depends mainly on the availability and cost of the material, method of construction, and purpose for which channel is to be used. The purpose of lining a channel is in most cases to prevent erosion, but occasionally it may be to check seepage losses. In lined channels, the maximum permissible velocity can be ignored provided that water does not carry sand, gravel or stones. The minimum permissible velocity is the lowest velocity that will not start sedimentation and induce the growth of aquatic plant and moss. The longitudinal bottom slope of a channel is generally governed by the topography and energy head required for flow of water. In many case, the slopes may depend also on purpose of channel. The freeboard of a channel is the vertical distance from the top of the channel to the water surface at the design condition. It must be sufficient to accommodate the waves and fluctuation in the water level. Freeboard varies from 5% to 30% of the depth of the flow. The relationship between width and depth varies widely. If a hydraulically efficient channel section is to be adopted then.

\[
\frac{B}{y^0} = 2\left( \sqrt{m^2 + 1} - m \right).
\]

PROBLEM 32.

Consider an open channel of rectangular cross-section, with bottom width of 6m, containing water flowing 2 m deep. The bottom slope of the channel is 0.0004 and it is made of concrete with a Manning roughness coefficient of 0.011. What would be the average flow velocity of the water and what would be the volumetric water flow rate?

Hyalenic radius \( R = A/P = (2)(6)/(6 + 2 + 2) = 1.2 \) m

\[ V = (1.49/0.011)(1.2^{1/3})(0.0004^{1/2}) = 2.37 \text{ m/sec} \]

\[ Q = VA = (2.37 \text{ m/sec})(12 \text{ m}^2) = 28.44 \text{ m}^3/\text{sec} \]

PROBLEM 233

Given a trapezoidal channel with bottom width of 3m, side slope 1.5H:1V, a longitudinal slope of 0.0016, and a roughness coefficient on N=0.013. Determine the normal discharge if the normal depth of flow is 2.6m.
Cross sectional area is given by,

\[ A = (b+ny)y = (3+1.5\times2.6)\times2.6 = 18\text{m}^2 \]

Wetted perimeter is given by,

\[ P = b+2y\sqrt{(1+m^2)} = 3+2\times2.6\times\sqrt{(1+1.5^2)} = 12\text{m} \]

Hydraulics radius,

\[ R = \frac{A}{P} = \frac{18}{12} = 1.5\text{m} \]

Manning’s formula,

\[ V = \left(\frac{R^2S}{n}\right) \]

Or, Discharge,

\[ Q = VA = \left(\frac{AR^2S}{n}\right) = 73\text{m}^3 \]

**PROBLEM 34**

A rectangular channel has a bottom slope of 0.0064, Manning’s coefficient, N=0.015 and a width of 6m. It carries a discharge of 6 cumecs. Compute the normal depth and the velocity.

Given: bed width, b=6.0m; bottom slope, S=0.0064; Manning’s coefficient, n=0.015; Discharge, Q=6 cumecs.

We know the relation,

\[ \frac{QN}{\sqrt{S}} = AR^2 \]

Upon substitution, we get,

\[ AR^2 = 1.125 \]

<table>
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<th>y</th>
<th>A=by</th>
<th>P=b+2y</th>
<th>R=A/P</th>
<th>R^2/N</th>
<th>AR^2/N</th>
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<td>1.124</td>
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</tbody>
</table>

Therefore, normal depth =0.384.

Velocity, \( V = (Q/A) = 6/2.304 = 2.60 \text{ m/s} \).
Expected questions

PART A

1. What is meant by normal depth?
2. State the condition for maximum discharge in circular channel.
3. What are the instruments used for measuring velocity in a river?
4. Differentiate between normal depth and alternate depth.
5. Find the critical depth of a rectangular channel carrying a discharge of 2.4 m³/s/m.
6. Define control section and how it affects the flow depth
7. Define most economical cross section and list the condition for a trapezoidal channel?
8. What is the significance of most economical section?
9. What is the classification of channel bottom slope which is used in flow profile computation?
10. How do you measure the velocity of flow in open channel?
PART B

1. A canal is formed with side slopes 2 : 1 and a bottom width of 3.0m. The bed slope is 1 in 4500. Using manning’s formula and assuming manning’s n as 0.025. Calculate the depth of water for a discharge of 3.0m3/sec for a uniform flow.

2. Determine the dimensions of the most economical trapezoidal channel with manning’s n = 0.02, to carry a discharge of 14m3/sec at a slope of 4 in 10,000.

3. Determine the longitudinal slope of a triangular channel carrying 1.2m3/sec for a normal depth of flow 0.75m and a side slope 2 : 1. Take chezy’s C = 45.

4. A trapezoidal channel with side slope 1 to 1 has to be designed to convey 10m3/sec at a velocity of a 2m/sec so that the amount of concrete lining for the bed and sides is the minimum. Calculate the area of lining required for one metre length of channel.

5. What diameter of a semicircular channel will have the same discharge has a rectangular channel of width 2.5m and depth 1.25m?. Assume the bed slope and Manning’s ‘n’ are the same for both the channels.

6. A canal is formed with side slopes 2:1 and a bottom width of 3.0m. The bed slope is 1 in 4500. using manning’s formula and assuming manning’s ‘n’ as 0.025, calculate the depth of water for a discharge of 3.0m3/sec for a uniform flow.

7. For the purpose of discharge measurement yht width of a rectangular channel is reduced gradually from 4.0m to 2.0m and the flow is raised by 0.45m at a given section. When the approaching depth of flow is 2m, what rate of will be indicated by a drop of 0.3m in the water surface elevation at the contracted section?

8. Obtain an expression for the depth of flow in a circular channel which gives maximum velocity for a given longitudinal slope. The resistance to flow can be expressed by manning’s equation.

9. In a rectangular channel 3.5m wide, flow depth of 2m, find how high can be raised without causing afflux. If the upstream depth of flow raised to 2.5m what should be the height of the hump? Flow in the channel is 26.67m3/sec.

10. Calculate the critical depth and corresponding specific energy for a discharge of 5.0m3/sec in the following channel.
   i) Rectangular channel of bedwidth 2.0m
   ii) Triangular channel of side slope 1h and .5v
   iii) Circular channel of diameter 2.0m

11. Prove that for maximum discharge in circular channel the depth of flow is equal to 0.95 times diameter of the channel.
12. A trapezoidal channel having bottom width 6m and side slope 2h and 1v is laid in the
bottom slope of 0.0016. If it carries a uniform flow of water at the rate of 10m3/sec,
compute the normal depth and the mean velocity of flow. Take manning's n as 0.025.

13. Define uniform flow in open channel and write chezy’s equation.

14. The trapezoidal channel of bottom width of 3m side slope 1.5h and 1v carries discharge
of 10m3/sec at a depth of 1.5m under uniform flow condition the longitudinal slope of
channel is 0.001. Compute manning’s roughness coefficient of the channel.

15. A circular pipe diameter 600mm carries discharge 0.2m3/sec will flow half full. Determine
the slope of pipe to be laid in the ground. Assume manning’s n=0.013 for concrete pipe.
Also determine the depth of flow if the pipe is laid in a slope of 0.01.

16. Derive chezy’s formulae to determine the velocity of flow in open channel.

17. Determine the discharge through a rectangular channel of width 2m having a bed slope
of 1 in 2000. The depth of flow is 1.5m and the value of manning constant n is 0.012.

18. Determine the dimensions of most economical trapezoidal channel section with 1.5 side
slope to carry 10 cumecs of water on a bed slope of 1 in 1600.

19. The rate of flow of water through a circular channel of diameter 0.6m is 0.15 cumecs.
Determine the slope of bed of the channel for maximum velocity. Assume c as 60.

20. Show that for a trapezoidal channel of a given area of flow, the condition of maximum
flow requires that hydraulic mean depth is equal to one half of the depth of flow.

21. The circular sewer 0.6m inner diameter has a slope of 1 in 400. Find the depth when the
discharge is 0.283m3/sec. Take c=50.

22. Define specific energy of flow at a channel section. Draw the specific energy curve and
explain.